# Midterm

International Trade I at ITAM - Prof. Tiago Tavares

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1. (30) Consider that an economy can produce food and clothing using only labor according to the following production functions:

$$\begin{aligned} y_c &= z_c l_c \\ y_f &= z_f l_f \end{aligned}$$

The representative consumer in that economy values consumption of clothing, $x_c$, and food, $x_f$, with an utility function of the form:

$$u(x_c, x_f) = \alpha \log x_c + (1 - \alpha) \log x_f$$

Moreover that consumer supplies a fixed amount of $L$ units of labor, freely mobile across industries.

   (a) Under autarky, solve for the consumer problem.

   (b) Still under autarky, solve for the firms' problem.

   (c) What is the equilibrium relative price of the final goods? And the real wage in term of clothing?

   (d) Assume that $L = 16$, $\alpha = 0.5$, and $z_c = z_f = 1$. Let's now add a trading partner with the same preferences and $L^* = 6$, $z_c^* = 4$, and $z_f^* = 2$. Characterize the new free-trade equilibrium as the relative prices and relative world supply.

   (e) What is the home economy new real wage rate (in terms of clothing)?

   (f) Suppose now that $z_f^* = 3$. What is the new relative final goods prices and home real wage rate (in terms of clothing)? What are the welfare impact in the home economy of such change in the foreign country's productivity?

2. (15) Suppose we have 2 countries each producing more than 2 goods, that were initially trading with each other. Suppose now that one of these two economies, call it foreign economy, improves its productivity uniformly across all the sectors. Describe in words what would happen to the home economy's welfare.

3. (20) Let's assume that we have a continuous of sectors indexed by $i \in [0,1]$. Let $z(i)$ and $z^*(i)$ be functions describing the productivity of both a home and a foreign economy that value, accordingly to the same Cobb-Douglas utility function, consumption in all sectors. Labor is the only factor of production in both economies. Assume that $Z(i) \equiv z(i)/z^*(i)$ is a decreasing function for all $i \in [0,1]$.

   (a) Show that in a competitive market we can only have one sector, $\hat{i}$, such that for all $i < \hat{i}$ production is located at home, while for $i > \hat{i}$ production is located at the foreign economy. (*hint: in equilibrium we must have that for any $i$, $p(i)z(i) - w \leq 0$ and the inequality holds with equality if $i$ is produced; you can start by assuming that if we have an $i' < i$ such that $i'$ is produced abroad and $i$ is produced domestically, then we reach a contradiction)*

   (b) We can show that an equilibrium is achieved by the intersection of 2 curves:

$$\frac{w}{w^*} = z(i)/z^*(i)$$
$$\frac{w}{w^*} = \frac{\theta(i)}{1-\theta(i)} \cdot \frac{L^*}{L}$$

   where $\theta(i)$ is the share of expenditure consumed in both countries on goods produced in the home economy. Please provide a graphical description of the equilibrium. Describe the intuition through which a new world trade equilibrium is reached if $L^*$ where to decrease.

   (c) Suppose we allow for a transfer (a donation of resources) from the foreign to the home economy equal to $T$, that is, the new income of the both economies become:

$$\begin{array}{ll} wL + T & \textit{home} \\ w^*L^* - T & \textit{foreign} \end{array}$$

   What are the consequences of that transfer in terms of the home economy terms of trade?

(d) Would your previous answer change if the home consumption was biased towards home goods, that is, if $\theta(i) > \theta^*(i)$? (*no math required here if you don't want*)

4. (20) Assume that input-output factor requirements coefficients are fixed. The table shows capital requirements per unit of output ($a_{kj}$) and labor requirements per unit of output ($a_{lj}$) to produce one unit of output from 1 through 3. Also shown is the prevailing world prices for each commodity:

| | Commodity $j$ | | |
|---|---|---|---|
| | 1 | 2 | 3 |
| $a_{kj}$ | 4 | 1 | 1 |
| $a_{lj}$ | 1 | 2 | 4 |
| price | \$16 | \$14 | \$16 |

Let the supply of capital be 1 while the supply of labor be 3.

(a) What will this economy produce in equilibrium (*hint: the economy will produce at most 2 goods*)? What are the wage and rent of capital? What will be the production levels?

(b) Suppose now that the price of good 3 increases to \$20. What are the new equilibrium wage and rent of capital? What will be the production levels?

(c) Can you relate the previous answer with any of the theorems that you know? Please give a informal statement of such theorem.

(d) Instead suppose that the price of good 1 changes to \$48. Which goods will be produced in equilibrium and what will be the wage and rental rate of capital?

(e) List 2 potential problems of testing the Heckscher–Ohlin model predictions against the empirical evidence.

5. (15) Consider a 2 country version of the Armington model where consumption utility is given by:

$$\begin{aligned} C_h &= \left[\alpha^{1/\sigma} x_{hh}^{(\sigma-1)/\sigma} + (1-\alpha)^{1/\sigma} x_{fh}^{(\sigma-1)/\sigma}\right]^{\sigma/(\sigma-1)} \\ C_f &= \left[\beta^{1/\sigma} x_{ff}^{(\sigma-1)/\sigma} + (1-\beta)^{1/\sigma} x_{hf}^{(\sigma-1)/\sigma}\right]^{\sigma/(\sigma-1)} \end{aligned}$$

This world economy has transportation costs $\tau_{ij} \geq 1$ implying that prices are given by (we are assuming that $z_f = z_h = 1$):

$$p_{ij} = w_i \tau_{ij}$$

for $i, j = h, f$, implying that $p_{ij}$ is the destination price in country $j$ of a good produced in country $i$. Moreover each country is endowed with a fixed amount of labor $L_h$ and $L_f$.

(a) Set up the consumer problem of the home economy. We can show that the demand for foreign goods is given by $x_{fh} = (1-\alpha)\left(\frac{p_{fh}}{P_h}\right)^{-\sigma} C_h$. Please derive a formula for $P_h$ as a function of prices (*hint: $P_h$ is equal to the Lagrangean multiplier of the expenditure minimization problem*).

(b) We can show that from that model we can represent a gravity equation given by:

$$p_{hf} x_{hf} = (\tau_{hf})^{1-\sigma} \cdot X_f X_h \cdot \frac{P_f^{\sigma-1}(1-\beta)}{\alpha\left(\frac{\tau_{hh}}{P_h}\right)^{1-\sigma} X_h + (1-\beta)\left(\frac{\tau_{hf}}{P_f}\right)^{1-\sigma} X_f}$$

where $X_i$ is the income level of economy $i$ and $P_i$ a general price index. Moreover this equation has been represented by an empirical counter-part given by:

$$T_{hf} = \frac{X_f X_h}{D_{hf}} \cdot A$$

Where $T_{hf}$ are exports of the home economy into the foreign economy, $D_{hf}$ is the distance between the home and the foreign economy, and $A$ is a residual (empirical term).

i. Can we interpret $D_{hf}$ in a broader sense than just distance? Name a few of such interpretations.

ii. Do you see any limitation of using the second equation to do predictions for the effects of a worldwide elimination of, say, tariffs?