Handout 3 - Implications and Extensions of the Ricardian Model

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1 Implications and extensions of the Ricardian model

Following notes in lecture 1, we assume that, under the 2 country 2 good Ricardian model, the domestic economy has a comparative advantage in the production of good 1:

$$\frac{z_1}{z_2} \geq \frac{z_1^*}{z_2^*}$$

where z_i^j is the labor productivity of good i in country j.

1.1 Absolute advantage and relative wages

From the simple 2 country 2 good Ricardian model one could conclude that it is the comparative advantage and not the absolute advantage that determines the patterns of trade across countries. This, however, doesn't mean that absolute advantage is unimportant. The reason is that productivity differences, that determine the absolute advantage, play an important role in determining relative wages, *i.e*, the wages of the domestic economy relative to the foreign economy.

So see this point, note that under full specialization (domestic producing good 1 while

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foreign producing good 2) the following wage conditions need to hold:

$$\begin{aligned} w &= p_1^T \cdot z_1 \\ w^* &= p_2^T \cdot z_2^* \end{aligned}$$

where p_1^T, p_2^T are the prices that prevail under free trade. The correspondent relative wage becomes:

$$\frac{w}{w^*} = \frac{p_1^T}{p_2^T} \cdot \frac{z_1}{z_2^*} \tag{1}$$

Note also that, under free trade, the relative price of good 1 is between the relative autarky prices that would prevail in both the domestic and foreign economy:

$$\frac{p_1}{p_2} = \frac{1/z_1}{1/z_2} \leq \frac{p_1^T}{p_2^T} \leq \frac{p_1^*}{p_2^*} = \frac{1/z_1^*}{1/z_2^*} \tag{2}$$

Putting together (1) and (2) and rearranging yields the following:

$$\frac{z_2}{z_2^*} \leq \frac{w}{w^*} \leq \frac{z_1}{z_1^*}$$

That is, the relative wage lies between the ration of the productivities in each sector. Under free trade, even if the domestic economy is not producing good 2 (without comparative advantage), a very large productivity in that same sector would imply a large relative wage. This shows that productivity differentials are important even in those sectors where an economy is not specializing as these determine the relative wages under trade.

1.2 Gains from trade

To see the welfare gain from free trade under the Ricardian model, it is sufficient to look at the real wages. This is the case since labor income is the only revenue available to households' consumption.

Note that under autarky, the domestic country has to produce on both sectors thus, from a zero profit condition, wages under autarky w^a must observe that:

$$z_2 p_2^a - w^a = 0$$

Similarly, because under free trade the domestic economy specializes on good 1, it must be that

$$z_2 p_2^T - w^T \leq 0$$

(note that if specialization is full, the inequality is strict). Substituting z_2 from the first equation into the inequality yields:

$$\frac{w^T}{p_2^T} \geq \frac{w^a}{p_2^a}$$

But this implies that, under free trade, real wages are larger than under autarky. Similar arguments allow us to conclude the same for the foreign economy.

The idea is that, by allowing for trade, labor is reallocated into those sectors where revenue at world market prices is higher thus pushing wages up.

1.3 Ricardian model extension: multiple goods

In the previous class we saw that in the 2 good model, comparative advantage implies:

$$\frac{1/z_1}{1/z_2} \leq \frac{1/z_1^*}{1/z_2^*}$$

$$\Rightarrow \quad \frac{1/z_1}{1/z_1^*} \leq \frac{1/z_2}{1/z_2^*}$$

These two good economy can be easily generalized into a multiple good one. Let be N goods and rebrand those goods such that comparative advantage is highest for sectors with a low index:

$$\frac{1/z_1}{1/z_1^*} \leq \frac{1/z_2}{1/z_2^*} \leq \cdots \leq \frac{1/z_i}{1/z_i^*} \leq \cdots \leq \frac{1/z_N}{1/z_N^*}$$

Given this multitude of goods the question is what goods are produced domestically and abroad. International competition determines that, given equilibrium wages, goods are produced where they are less expensive. For example, good i would be produced in the domestic economy if:

$$w \cdot 1/z_1 \leq w^* \cdot 1/z_1^* \tag{3}$$

where w is the wage rate and $1/z_1$ is the labor requirement to produce one unit of good 1. Rearranging the above equation gives us a criterium about the goods that are produced at home. Essentially, a good i is produced at home if

$$\frac{w}{w^*} \leq \frac{z_i}{z_i^*}$$

The following assumptions will help us to determine the equilibrium $\hat{i}$ such that bellow $\hat{i}$ production occurs at home and the relative wage w/w^*.

Let $N \to \infty$ and define a continuum of goods indexed by $i \in [0,1]$. This implies that, for example, home labor productivity of good i is $z(i)$. From the previous discussion, goods with a low i index have a higher comparative advantage. That is for $j > i$

$$\frac{1/z(i)}{1/z^*(i)} \leq \frac{1/z(j)}{1/z^*(j)}$$

Note that for any i, profit maximization implies that

$$p(i)z(i) - w \leq 0 \qquad \text{(equality if i is produced at home)}$$
$$p(i)z^*(i) - w^* \leq 0 \qquad \text{(equality if i is produced at foreign)}$$

Given this structure it can be shown that there is a $\hat{i}$ such that production at home occurs with $i < \hat{i}$ while for $i > \hat{i}$ it occurs abroad. But this implies that at $\hat{i}$, both countries are indifferent between producing or not:

$$p(\hat{i})z(\hat{i}) - w = 0 \tag{4}$$
$$p(\hat{i})z^*(\hat{i}) - w^* = 0 \tag{5}$$

Dividing one equation into the other implies:

$$\frac{w}{w^*} = \frac{1/z^*(\hat{i})}{1/z(\hat{i})} \equiv 1/A(\hat{i}) \tag{6}$$

where $A(i) \equiv \frac{1/z(i)}{1/z^*(i)}$ is, by definition, an increasing function (reflecting decreasing comparative advantage).

From the demand side of the economy we assume that the consumer utility function is characterized by a Cobb-Douglas function with constant expenditure share $b(i)$ such that:

$$\int_0^1 b(i)di = 1$$

Given this utility function, demand of good i is characterized by being a constant share of real income:

$$p(i)x(i) = b(i) \cdot wL$$

Assuming that preferences are the same in the foreign and domestic economy one has that, for each country, the expenditure share on domestically produced goods to be equal to:

$$\theta(\hat{i}) = \int_0^{\hat{i}} b(i)di$$

But then trade balance implies that exports must equate imports:

$$\theta(\hat{i})w^*L^* = \left[1 - \theta(\hat{i})\right] wL \tag{7}$$

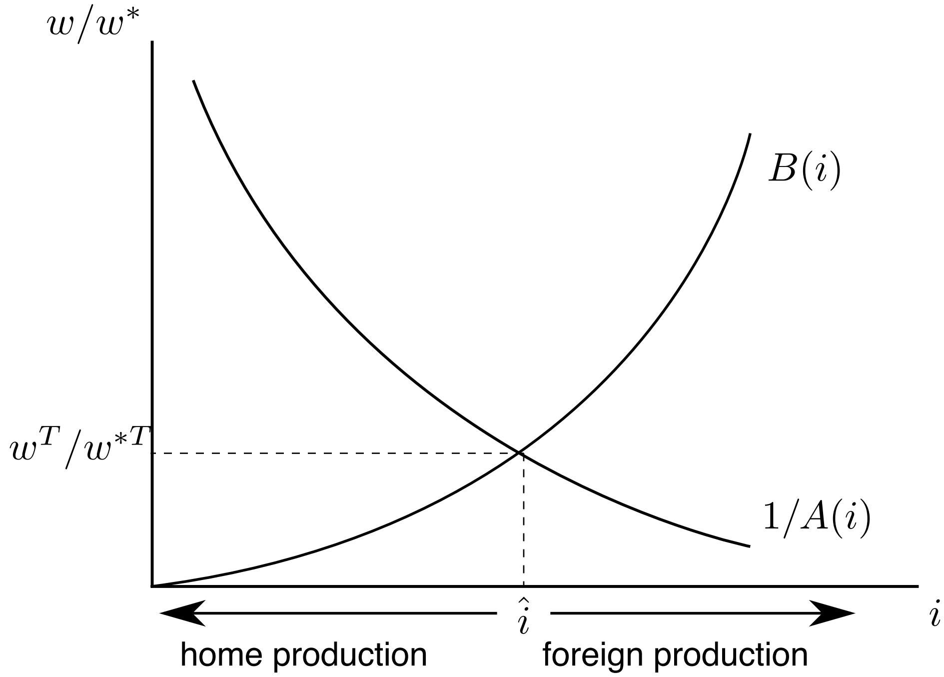
where the LHS of the equation are the exports of the home economy and the RHS the exports. Rearranging equation (7) yields:

$$\frac{w}{w^*} = \frac{\theta(\hat{i})}{1 - \theta(\hat{i})} \frac{L^*}{L} \equiv B(i) \tag{8}$$

And $B(i)$ is an increasing function (an increase in $\hat{i}$ would imply a trade surplus of the home economy, requiring a higher wage at the domestic economy to equilibrate the market).

Because equation (6) is decreasing while (8) is increasing, the equilibrium variables w/w^* and $\hat{i}$ can be determined. This can be easily seen in the following figure.

Figure 1: Equilibrium in a Ricardian model with multiple goods



In this example, specialization is complete with the domestic economy producing all goods at the left of $\hat{i}$ while the foreign economy produces all the goods at the right of $\hat{i}$.

Comparative statics analysis is particular simple in this model. Using equations (6) and (8) it is easy to see that, for example, an increase in the foreign labor supply L^* implies an increase in w/w^* and a decrease in $\hat{i}$; or a shift up of foreign productivity in all sectors $z^*(i)\forall i$ implies a fall in w/w^* and $\hat{i}$.

1.4 Trading costs in the Ricardian Model

Suppose now that for each unit that is exported abroad, only a fraction $1/\gamma$ arrives to the destination where $\gamma > 1$. These type of costs, colloquially named iceberg costs, can represent depreciation of goods, transportation costs, or other barriers to commerce. If a country exports x units at price $p(i)$, but only $1/\gamma$ units arrive at the destination, the cost for the importer becomes $p(i)/\gamma$. This implies that the effective price of exports are now:

$$p^{export}(i) = p(i)/\gamma$$

Using this on equations (4) and (5) imply that goods exported from home:

$$\frac{p(\hat{i})}{\gamma} z(\hat{i}) - w = 0$$
$$p(\hat{i}) z^*(\hat{i}) - w^* = 0$$

But then, equation (6) is shift down by the iceberg cost factor:

$$\frac{w}{w^*} = \frac{1/z^*(\hat{i})}{1/z(\hat{i})} \cdot \frac{1}{\gamma} \equiv 1/A^{exp}(\hat{i})$$

and because $A(i) = z^*(i)/z(i)$ and $A^{exp}(i) = \gamma z(i)^*/z(i)$, $A^{exp}(i) > A(i)$. While for goods that are imported from the home economy:

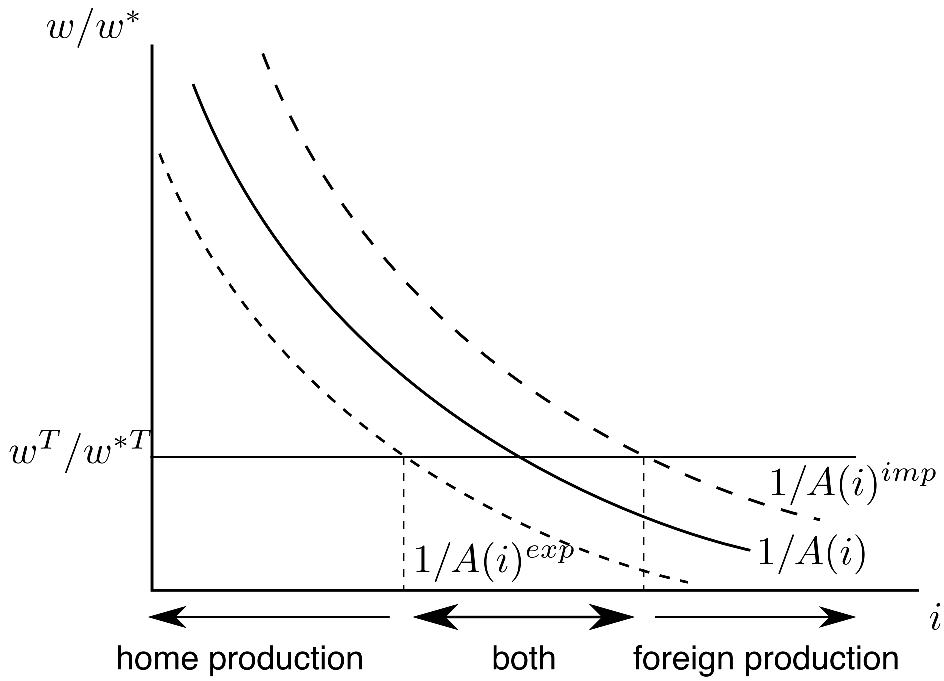
$$\frac{w}{w^*} = \frac{1/z^*(\hat{i})}{1/z(\hat{i})} \cdot \gamma \equiv 1/A^{imp}(\hat{i})$$

and $A^{imp}(i) < A(i)$. Note that for any i, $A^{exp}(i) > A(i) > A^{imp}(i)$. This implies that for a particular relative wage w/w^* there will be a no specialization region defined by:

$$\left\{ i : 1/A^{exp}(i) < w/w^* < 1/A^{imp}(i) \right\}$$

The equivalent graphical representation can be seen in the next figure where under the region 'both', both countries are producing those goods, which means that specialization is incomplete.

Figure 2: Iceberg costs in a Ricardian model with multiple goods



To understanding the meaning of the above equations recall that from our production criteria (3), output in sector i is produced at home if it's cheaper to do so, that is:

$$w\frac{1}{z(i)} \leq w^*\frac{1}{z^*(i)}\gamma$$

where we need to include in the RHS the iceberg cost $\gamma > 1$ since the indifference should be in producing at home or importing from abroad where, to get one unit of the good at the destination, only γunits have to be produced at the origin to accommodate for transportation losses. Rearranging the previous equation yields:

$$\frac{w}{w^*} \leq \frac{z(i)}{z^*(i)}\gamma \equiv \frac{1}{A^{imp}(i)}$$

which is precisely the region to the left of the intersection of A^{imp} with w/w^* in the graph.

Further more, if we take logs of the above equation:

$$\begin{aligned} & \log w - \log w^* \leq \log z(i) - \log z^*(i) + \log \gamma \\ \Rightarrow \quad & \log z^*(i) - \log z(i) \leq \log w^* - \log w + \log \gamma \end{aligned}$$

Which has the following approximated interpretation:

$$\%diff\, productivity\, btw\, F\, and\, H \leq \%diff\, wages\, btw\, F\, and\, H + \%loss\, transportation$$

That is, in order for production to be located at home, it must be that the percentage difference between foreign relative to home, is smaller than the percentage difference in wages between the two countries to which we should add up the percentage lost due to transportation.