Handout 4 - The Specific Factors and Heckscher-Ohlin Models

Tiago Tavares*
International Trade I at ITAM

February 3, 2016

1 Heckscher-Ohlin and specific factor model

Contrary to what we assumed in the Ricardian model, we now assume that, in a two good economy, production in each sector other inputs besides labor. We will see that this new technological structure implies that the opportunity cost of producing one good in terms of the other is no longer constant for different output choices. Two common approaches either assume that these other factors are fixed in each sector, that is, these factors are not mobile across sectors, or an alternative where all factors are mobile across sectors. It should be noted that, across countries only trade in goods is available, but not mobility of factors. It turns out that, under certain conditions, trade in goods is enough to replicate free mobility of factor inputs across countries over which world production would be maximized.

1.1 Specific factor model

This model, also know as Ricardo-Viner, assumes two goods and three factors. We say say two of those factors are specific for each sector in the sense that they are not allowed to freely move across sectors.

Formally, we have two goods $i = 1, 2$; three factor L, K_1, K_2 where L is mobile across sectors and K_1, K_2 are specific in sector 1 and 2, respectively. Technology is characterized

*Email: Please contact me to tgstavares@gmail.com if you find any errors in the text.

by production functions that are homogeneous of degree one, that is, we observe constant returns to scale to all factors:

$$y_i = f^i(K_i, L_i);\ f^i\, has\, CRS;\ \frac{\partial f^i(K_i, L_i)}{\partial L_i} > 0\ \ and\ \ \frac{\partial^2 f^i(K_i, L_i)}{\partial L_i^2} < 0,\ \ i = 1, 2$$

To get some important properties of the model lets assume that goods' prices are given: p_1, p_2 (this would be the case under a small open economy). As usually, profit maximization implies:

$$p_i f_l^i \;=\; w\ \ \forall i = 1, 2 \tag{1}$$

$$p_i f_{k_i}^i \;=\; r_i\ \ \forall i = 1, 2 \tag{2}$$

where the notation $f_l^i = \partial f^i / \partial l$ is used. And labor market clearing:

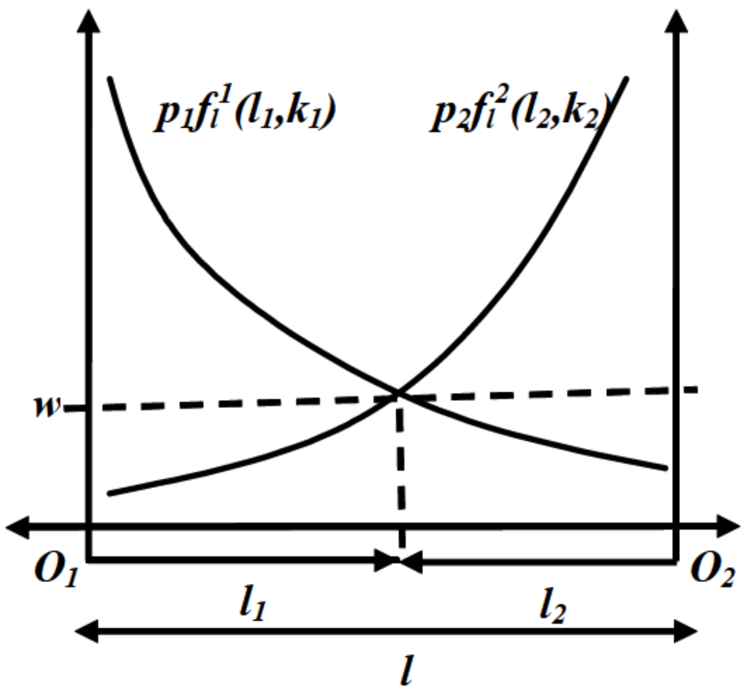
$$L = L_1 + L_2 \tag{3}$$

Note that (1), (2), (3), and p_1/p_2 is all that we need to solve for this economy's equilibrium. In particular using (1) and (3) we get the optimal labor allocation across sectors:

$$p_1 f_l^1(K_1, L_1) = p_2 f_l^2(K_2, L - L_1)$$

This can easily be seen using the following diagram of figure 1.

Figure 1: Optimal allocation of labor across sectors



1.2 Comparative statics in the specific factor model

Using figure 1, comparative statics is particular simple in this model. For example an exogenous terms of trade shock such that $(p_1/p_2)\uparrow$ implies that the curve $p_1 f_l^1$ in the graph increases. But this would imply that $L_1\uparrow$ and $w\uparrow$ as now there is more revenue to be made in sector 1. Similarly, using (2) we can conclude that $r_1/p_1\uparrow$ and $r_2/p_2\downarrow$. That is the real return for holders of K_1 increases while decreases for holders of K_2. Note that these returns shifts can also have an impact on the welfare of different agents of the economy (workers, and owners of K_1 and K_2), implying some winers and losers from these type of shocks.

Other cases can also be studied. It will be left as an exercise the impact of trade from an increase in, say K_1 or L.

1.3 The Heckscher-Ohlin model

Contrary to the previous model we know assume that all factors are freely mobile across sectors. To keep things simple we assume two goods ($i = 1, 2$) and two factors of production

(L, K for labor and capital). The technology of this economy is characterized by production functions with constant returns to scale:

$$y_i = f^i(K_i, L_i);\; f^i\, has\, CRS;\; \frac{\partial f^i(K_i, L_i)}{\partial L_i} > 0\;\, and\;\, \frac{\partial^2 f^i(K_i, L_i)}{\partial L_i^2} < 0,\;\; i = 1, 2$$

and both capital and labor are supplied in a fixed amount implying:

$$\begin{aligned} K &= K_1 + K_2 \\ L &= L_1 + L_2 \end{aligned}$$

To solve this model note that from duality in economics, it is possible to characterize the firms decisions using cost functions (where the dual of this problem would use profit functions). We will see that using costs functions substantially simplifies our results and will help us to derive some important results from the model. It follows that the cost minimization problem for a firm is given by:

$$C_i(w, r) = \min_{l_i, k_i} \left\{ wl_i + rk_i\;\, subject\, to\;\; f^i(k_i, l_i) \geq 1 \right\}$$

the solution of these problem will give us factor demand functions (with factor prices as arguments) that are required to produce one unit of output $y_i = 1$, that is, $l_i(w, r)$ and $k_i(w, r)$ are unit demand functions of factors l and k required in the production of i. Similarly note that $C_i(w, r)$ give us the cost required to produce one unit of output i. Note that the derivative of the cost function with respect with with the cost functions is just equal to the factor demand equations (this is an application of the envelope theorem):

$$\begin{aligned} \frac{\partial C_i(w, r)}{\partial w} &= l_i(w, r) \\ \frac{\partial C_i(w, r)}{\partial r} &= k_i(w, r) \end{aligned}$$

Let's assume for now that we are in a small open economy such that p_1, p_2 are exogenously given. To check the equilibrium conditions of this economy note that profit maximization implies:

$$p_i \leq C_i(w,r) \qquad \text{(i=1,2)}$$
$$p_i = C_i(w,r) \qquad \text{(if i is produced in equil)}$$

While market clearing implies that:

$$\begin{aligned} L &= L_1 + L_2 \\ K &= K_1 + K_2 \end{aligned}$$

or, equivalently, using our unit factor demand equations:

$$\begin{aligned} L &= y_1 l_1(w,r) + y_2 l_2(w,r) \\ K &= y_1 k_1(w,r) + y_2 k_2(w,r) \end{aligned}$$

1.3.1 Factor price equalization

Given the economy outlined in the previous section, an important question is if trade in goods can be a (perfect) substitute for trade in factors. To unequivocally answer this question we need to assume that factor intensity reversals are not allowed:

Assumption 1. *Factor Intensity Reversal (FIR) is not allowed:*

$$\frac{l_1(w,r)}{k_1(w,r)} \begin{matrix} > \\ < \end{matrix} \frac{l_2(w,r)}{k_2(w,r)} \quad \forall (w,r)$$

This assumption states that the relative intensity utilization of inputs cannot invert for different factor prices. In other words, if one sector is relatively more intense in labor than the other for a particular $(\bar{w},\bar{r})$, then it is also more intense in labor for any other choice (w,r). This assumption allow us to derive our first main result.

Theorem 1. *(Factor Price Insensitivity FPI) If both goods are produced in equilibrium and no FIR is allowed, then* (p_1,p_2) *uniquely determine* (w,r)

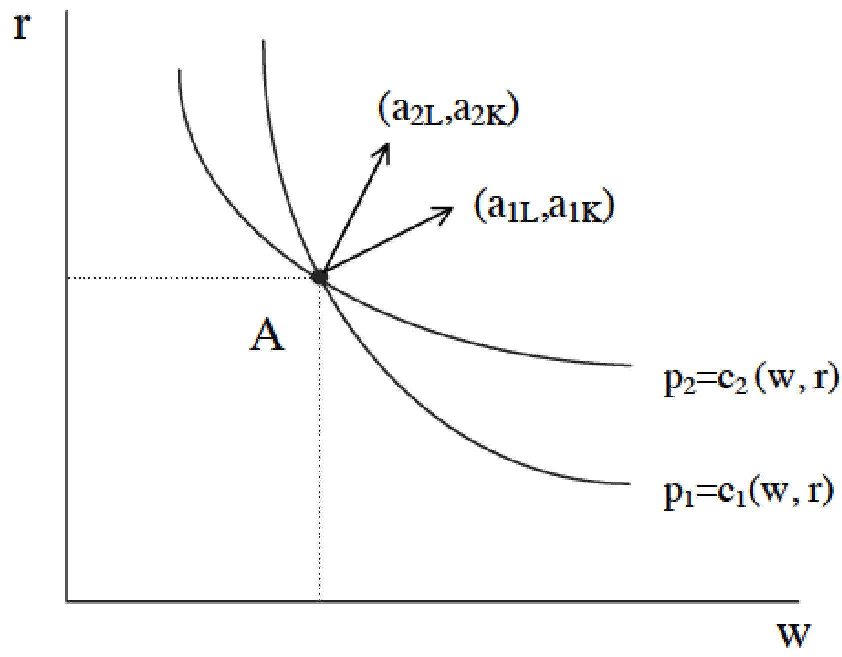
Several comments are in order:

- good prices rather than factor endowments determine factor prices
- this is not the case for a closed economy, but only prevails when good prices are exogenous

- not allowing for FIR is fundamental for our result

To see this last point consider the following 2 figures where $a_{iL} = l_i$ and $a_{ik} = k_i$.

Figure 2: No FIR

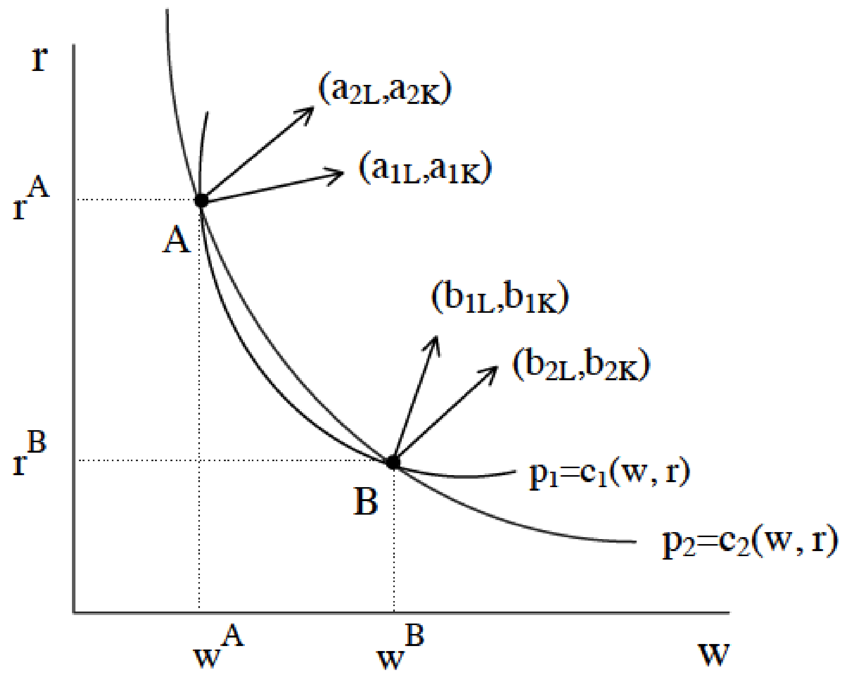


In figure 2 we plot the two cost functions (that in equilibrium must equal p_i) for sectors 1 and 2 in a (r, w) axis. Note that the slope of these functions gives us a vector pointing to (l_i, k_i)as:

$$\frac{\frac{\partial C_i(w,r)}{\partial w}}{\frac{\partial C_i(w,r)}{\partial r}} = \frac{l_i\left(w, r\right)}{r_i\left(w, r\right)}$$

Because the curves only intersect at one point A, it must be that $l_2/k_2 < l_1/k_1$ so FIR doesn't hold in this economy.

Figure 3: No Fir violation



But this is not the case in figure 3. Now the cost functions intersect twice implying that in point B we have $l_2/k_2 < l_1/k_1$, while in point A the opposite occurs $l_2/k_2 > l_1/k_1$. This violation of no FIR implies that for the same prices (p_1, p_2) two different factor prices can emerge (points A and B).

To finalize this section note that we still need to relate endowment (K, L) values with the economy's output (y_1, y_2). Recall that market clearing implies:

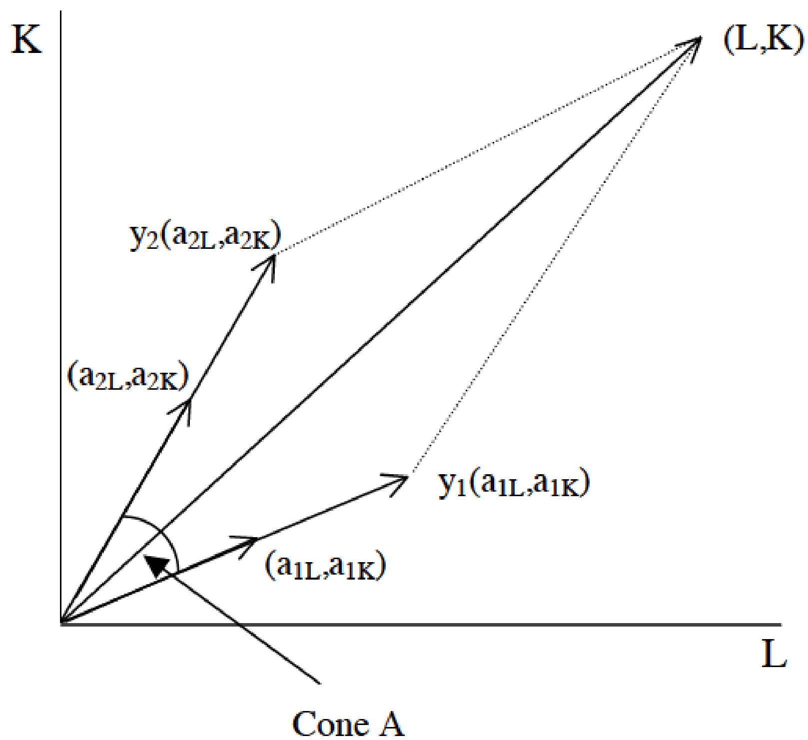
$$\begin{aligned} L &= y_1 l_1 (w, r) + y_2 l_2 (w, r) \\ K &= y_1 k_1 (w, r) + y_2 k_2 (w, r) \end{aligned}$$

or, in matrix notation:

$$\begin{pmatrix} L \\ K \end{pmatrix} = y_1 \begin{pmatrix} l_1 (w, r) \\ k_1 (w, r) \end{pmatrix} + y_2 \begin{pmatrix} l_2 (w, r) \\ k_2 (w, r) \end{pmatrix}$$

It follows that for any endowment (L, K), there will be a unique value for outputs (y_1, y_2) such that when (l_1, k_1) and (l_2, k_2) are multiplied by these amounts, they will sum to these endowments. The illustration of this step can be seen in figure 4

Figure 4: Cone of specialization to illustrate output choices



As a direct implication of the previous theorem, a slightly stronger result is known as the factor price equalization theorem is stated in the following way:

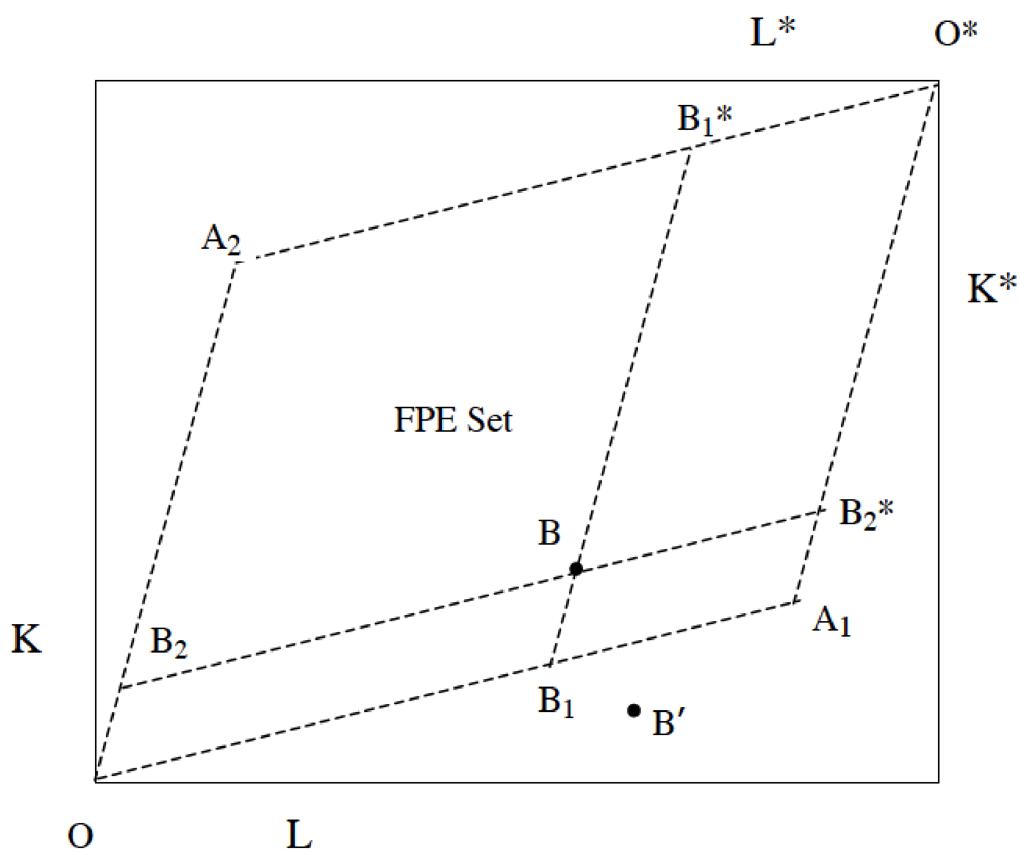
Theorem 2. *(Factor Price Equalization FPE - Samuelson 1949) If two countries produce both goods under free trade with the same technology and FIR doesn't occur, then they must have the same factor prices*

As with before, several comments are in order:

- trade in goods can be a perfect substitute for trade in factors
- countries with different factor endowments can sustain the same factor prices through different allocation of factors across sectors

To illustrate how this theorem is proved, we first assume that labor and capital is free to move between the two countries until their factor prices are equalized. Then all that matters for factor prices are world endowments of labor and capital (in figure 5 those are represented by the length of the horizontal and vertical axis). Note that given factor prices (implied by the slope of the segments (OA_2) and (OA_1), world output is given by the length of (OA_1) and (OA_2). Now consider that factor endowments are not mobile across countries, so we have a particular allocation, say B. It's easy to see that, for the same factor prices (or slopes), there are unique specialization cones such that the same world output is achieved. That is: $(OA_2) = (OB_2) + (O^*B_2^*)$ and $(OA_1) = (OB_1) + (O^*B_1^*)$. These arguments essentially prove the previous theorem.

Figure 5: Two country trade equilibrium and factor price equalization



Using the same model we can still show two additional important results regarding the implication of changes in the terms-of-trade to real returns (Stolper-Samuelson theorem) and on changes endowments to each sector output (Rybczynski theorem).

Theorem 3. *(Stolper-Samuelson theorem, 1941) An increase in the relative price of a good* *will increase the real return to the factor used intensively in that good, and reduce the real return to the other factor*

Proof. Without loss of generality let $l_1/k_1 > l_2/k_2$ and $\hat{p}_2 \equiv \frac{dp_2}{p_2} > \hat{p}_1 \equiv \frac{dp_1}{p_1}$. Note zero profits imply (for p_1, p_2 fixed):

$$p_i = c_i(w, r)$$

thus, total differentiation implies:

$$\begin{aligned} dp_i &= l_i dw + k_i dr \\ \Rightarrow \frac{dp_i}{p_i} &= \frac{wl_i}{c_i}\frac{dw}{w} + \frac{rk_i}{c_i}\frac{dr}{r} \\ \Rightarrow \quad \hat{p}_i &= \theta_{il}\hat{w} + \theta_{ik}\hat{r} \end{aligned} \tag{4}$$

and note that $\theta_{ik} = (1 - \theta_{il})$. Because of $l_1/k_1 > l_2/k_2$, $\theta_{l1} > \theta_{l2}$. From (4) we must have that $\hat{w} > \hat{p}_1$ and $\hat{r} < \hat{p}_2$ or $\hat{w} < \hat{p}_1$ and $\hat{r} > \hat{p}_2$ But then $\hat{r} > \hat{w}$. It follows that:

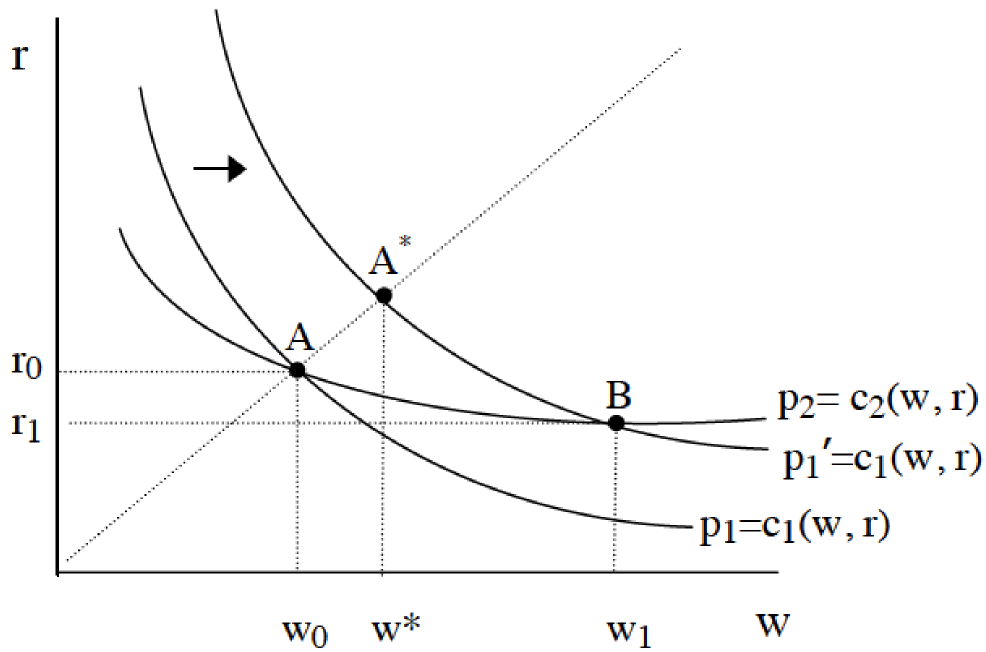
$$\hat{r} > \hat{p}_2 > \hat{p}_1 > \hat{w}$$

□

The comments that follow are:

- SS predicts both winners and losers from changes in relative prices
- In the empirical literature, people often talk about Stolper-Samuelson effects whenever looking at changes in relative factor prices (though changes in relative good prices are rarely observed)
- the illustration of this theorem can be found in the figure 6

Figure 6: Stolper-Samuelson theorem for an increase in p_1



Theorem 4. *(Rybczynsky, 1965) An increase in factor endowment will increase the output of the industry using it intensively, and decrease the output of the other industry*

Proof. Without loss of generality let $l_1/k_1 > l_2/k_2$ and $\hat{K} > \hat{L}$. Note factor market equilibrium imply (for p_1, p_2 fixed):

$$L = y_1 l_1 + y_2 l_2$$

thus, total differentiation implies:

$$dL = l_1 dy_1 + l_2 dy_2$$
$$\Rightarrow \frac{dL}{L} = \frac{y_1 l_1}{L}\frac{dy_1}{y_1} + \frac{y_2 l_2}{L}\frac{dy_2}{y_2}$$
$$\Rightarrow \quad \hat{L} = \lambda_{l1}\hat{y}_1 + \lambda_{l2}\hat{y}_2 \tag{5}$$
$$\hat{K} = \lambda_{k1}\hat{y}_1 + \lambda_{k2}\hat{y}_2 \tag{6}$$

and note that $\lambda_{l2} = (1 - \lambda_{l1})$. Because of $l_1/k_1 > l_2/k_2$, $\lambda_{l1} > \lambda_{k1}$. From (5) we must have that $\hat{y}_1 > \hat{L}$ and $\hat{y}_2 < \hat{K}$ or $\hat{y}_1 < \hat{L}$ and $\hat{y}_2 > \hat{K}$. But then $\hat{y}_2 > \hat{y}_1$. It follows that:

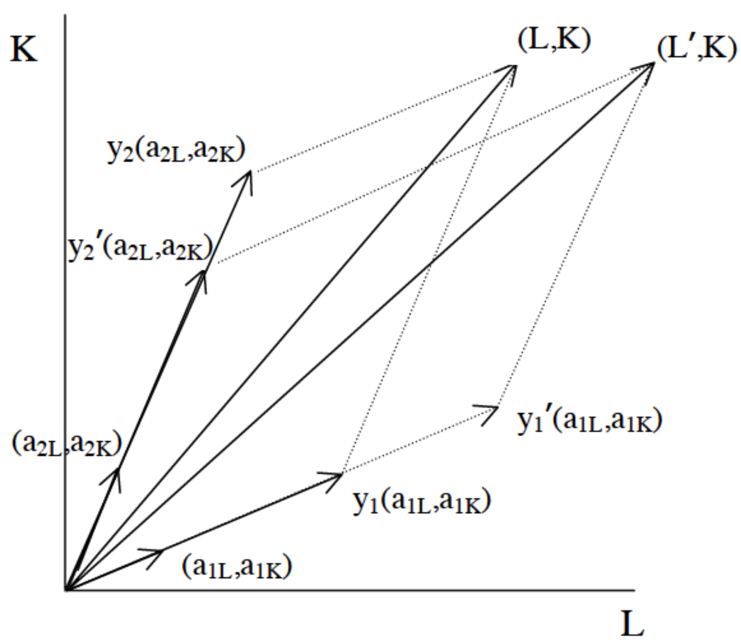
$$\hat{y}_2 > \hat{K}_2 > \hat{L}_1 > \hat{y}_1$$

□

Comments:

- for (p_1, p_2) given $\Rightarrow$ factor prices and factor requirements are not affected by changes in endowments
- the reason is because output will expand just enough to keep these factor prices constant
- empirically, Rybczynsky theorem suggests that impact of immigration may be very different in closed vs. open economy
- the illustration of this theorem can be found in the figure 7

Figure 7: Stolper-Samuelson theorem for an increase in L



1.3.2 Heckscher-Ohlin Theorem

Now, let us endogeneize goods prices in a model with two countries, two goods, and two factors (2x2x2). In order to do that let's assume that the home country is relatively labor abundant compared with the foreign country:

$$L/K > L^*/K^*$$

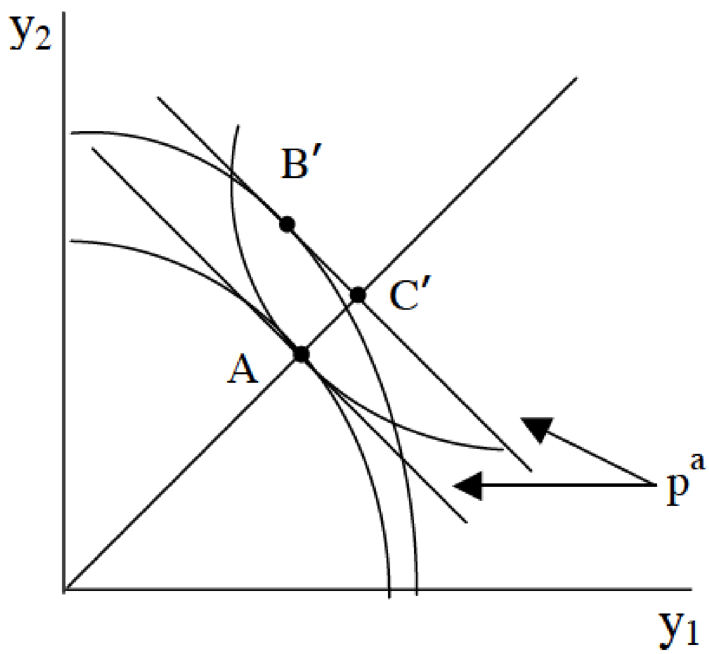
Moreover lets assume that both countries share the same technology and preferences with preferences being homothetic.

Given these assumptions, the Heckscher-Ohlin theorem states the following:

Theorem 5. *(Heckscher-Ohlin) Each country exports the good that uses its abundant factor intensively*

To prove this result lets start by determining the autarky prices in both economies and then the free-trade prices, with the correspondent patterns of specialization. Without loss of generality lets assume that $L = L^*$ and $K < K^*$ so the foreign economy is abundant in capital. Figure 8 illustrates the determination of autarky prices. Note that the foreign economy is represented by the PPF that drawn to the outside of the PPF of the domestic economy since it has more capital. The autarky equilibrium is given by a tangent of home's indifference curves and its PPF in point A with respective prices of $p = p_1/p_2$. Using the same price level in the foreign economy yields a tangency point at B' (which would observe the Rybczynsky theorem). However, at prices p the optimal choice of the foreign economy would be on C' (due to homothetic preferences). The difference between C' and B' generates an excess demand for good 1 that is, in general, resolved by an increase in prices. This argument suffices to show that $p^* > p$. That is in autarky prices in the foreign economy are larger than in the domestic economy.

Figure 8: Autarky prices for domestic and foreign economy



Letting $z(p)$ and $z^*(p)$ be a domestic and foreign excess demand function for good 1, we already shown that :

$$z(p) + z^*(p) > 0$$

Similar arguments can be used that world excess demand evaluated at autarky prices are:

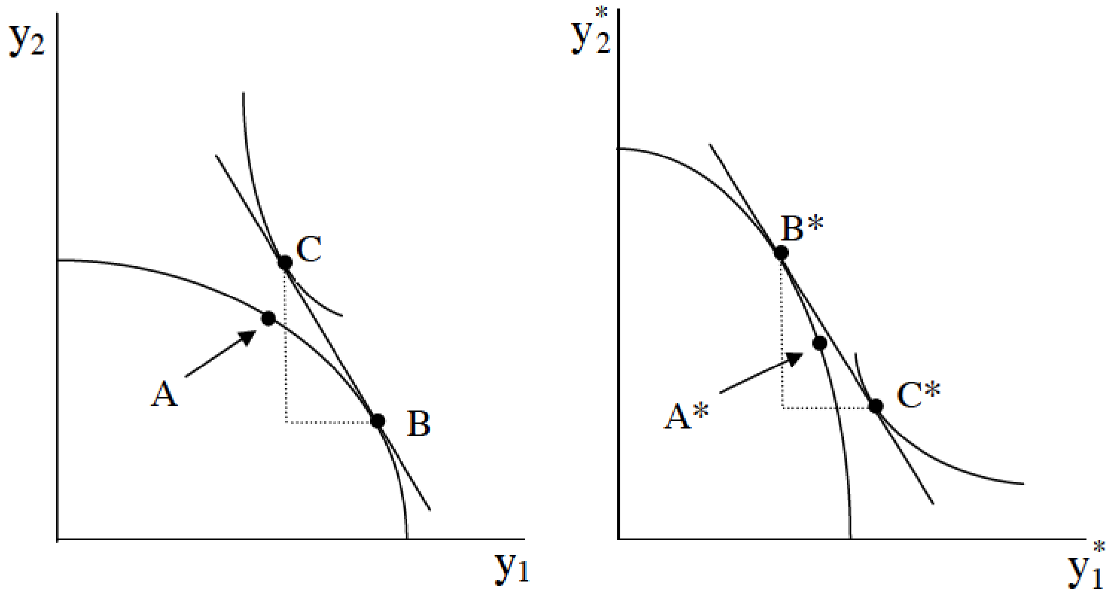
$$z(p^*) + z^*(p^*) < 0$$

Because exports need to be equal to imports, world excess demand must be zero. Using a continuity argument, we can conclude that

$$p^* > p^T > p$$

that is, free trade prices are within autarky prices. It is now easy to see that home country will export good 1 while the foreign country will export good 2 as illustrated in figure 9 where the left panel represents home while the right panel represents the foreign economy.

Figure 9: Patterns of specialization under free-trade



Note that in the above case, using the Stolper-Samuleson theorem, we can also conclude that free-trade will increase the return in the abundant factor, while decrease the return of the relative scarce factor.