Handout 9 - Intra-industry Trade and the Krugman Model

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1 Firm heterogeneity and trade

The classical trade models we've seen so far (Ricardian, specific factors, Heckscher-Ohlin) focus essentially on aggregate trade flows across countries, overlooking much of the incentives that occur at the firm level. Lowering the level of analysis to the firm has the potential benefit of accounting for many recently documented firm-level facts such as:

- within each industry there is firm-level heterogeneity
- fixed costs matter in export decisions
- within the same industry more productive firms are more likely to export
- trade liberalization leads to firm reallocation within each industry, correlated with export status

Melitz [2003] provides a model that can account for all these facts. The main blocks of such model are increasing returns to scale and love-for-varieties as in Krugman [1979] andKrugman [1980]with firms' market entry and exiting as in Hopenhayn [1992].

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1.1 The Krugman's model under autarky

The Krugman [1979] andKrugman [1980] models introduced two new motivation for trade, namely, *love for variety* and *increasing returns to scale*. By opening up to trade firms within a country are able to increase the scale of production thus reducing their average costs and, at the same time, consumers have access to a wider variety of goods. These features provides additional gains to trade. Moreover, this theory provides a rational on why we observe so much intra-industry trade across countries.

We'll start by describing the model equilibrium for an autarky economy and then analyze the new equilibrium once trade is allowed.

Household Let an economy have a fixed supply of L workers, each supplying a single unit of labor. Also, each worker enjoys consumption of N varieties of the same good[1] accordingly to the following, by now, familiar utility function:

$$C = \left(\sum_{i=1}^{N} c_i^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \tag{1}$$

This is a constant elasticity of substitution (CES) function[2] where $\sigma > 1$ is the elasticity of substitution and c_i is the level of consumption of variety i. Because each coefficient associated with the consumption of each variety c_i is the same, we say that the consumer values all varieties equally. It is important to stress that each variety i belongs to the same sector. Later in this note, we will interpret the trade of varieties that emerges in equilibrium between different countries as intra-industry trade, that is, trade of varieties within the same sector. We should also point that, contrary to the Armington model, the total number of varieties N will be an endogenous variable to be determined in equilibrium (more about that later). In particular, a consumer becomes better off, with the increase in the number of varieties. This property, that comes from the fact that $u(c_i) = c_i^{\frac{\sigma-1}{\sigma}} \Rightarrow u'(c_i) > 0 \text{ and } u''(c_i) < 0$, is usually known as *love-for-varieties*.

As usual, the problem for each worker consists in minimizing her expenditure subject

[1] These varieties may be also interpreted as brands, or different products within the same sector, for example, Coca-cola and Pepsi inside the soda sector.

[2] Recall that we already used this utility function in the Armington model.

to a particular utility level:

$$\min_{\{c_i\}_{i=1}^N} \left\{ \sum_{i=1}^N p_i c_i \quad st \quad \left(\sum_{i=1}^N c_i^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \geq \bar{C} \right\}$$

First order conditions imply the following demand functions for each $i = 1, ..., N$:[3]

$$c_i = \left(\frac{p_i}{P} \right)^{-\sigma} C \tag{2}$$

where

$$P = \left(\sum_{i=1}^N p_i^{1-\sigma} \right)^{\frac{1}{1-\sigma}} \tag{3}$$

$$C = \left(\sum_{i=1}^N c_i^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \tag{4}$$

Finally, each worker is constraint to her budget constraint, meaning that, individually, expenditure must equal income:

$$\sum_{i=1}^N p_i c_i = w + \pi \tag{5}$$

Where w is the wage rate per unit of labor and $\pi$is a share of the aggregate profits. Here π is a new element that must be introduced due to the possibility of the existence of firms profits, where those firms are owned by the workers. Thus profits should also be included in the workers income side of the budget constraint. Turns out that in equilibrium profits will be equal to zero so we will need to pay mostly attention to w (more about this result later).

Firms An important feature introduced in this model is the assumption that firms face increasing returns to scale. Suppose, for example, that variety i can be produced using only labor accordingly to the following production function:

$$q_i = \varphi l_i - \varphi f \tag{6}$$

[3] For the derivations of the demand functions, refer to the Armington model handout.

Note that the first term in the right hand side (RHS) of equation (6) has the usual meaning of a liner technology using labor with marginal productivity equal to φ. However, now there's an additional term φf which can be interpreted as an 'fixed cost' that has to be made irrespectively of the amount of labor used in the production. To see this more clearly, re-write equation (6) in terms of labor requirements to produce q_i units of variety i:

$$l_i = q_i/\varphi + f \tag{7}$$

It is clear from this equation that in order to produce whatever level of output q_i, a fixed amount of labor f has to be employed. Suppose that the wage rate equals $w = 1$, then the marginal and the average cost for the firm producing i would be:

$$\begin{aligned} MC(q_i) &\equiv \frac{\partial l_i}{\partial q_i} = \frac{1}{\varphi} \\ AC(q_i) &\equiv \frac{l_i}{q_i} = \frac{1}{\varphi} + \frac{f}{q_i} \end{aligned}$$

Two comments emerge from the cost structure of the firm. The first one is that the firm average cost of the firm is decreasing with the level of production. This is essentially why we say that the firm has increasing returns to scale: as the firm increase the scale of production, the productivity is higher as there is a larger amount of output to amortize the fixed cost. The second comment is that, due to the presence of the fixed cost, the average cost is always larger than the marginal cost: $AC(q_i) > MC(q_i)$. Under these conditions, the typical perfect competition price equal to marginal cost rule $P = MC(q_i)$ would imply negative profits:

$$\begin{aligned} & p_i = MC(q_i) < AC(q_i) \\ \Rightarrow \quad & q_i p_i = q_i MC(q_i) < q_i AC(q_i) = C(q_i) \\ \Rightarrow & R(q_i, p_i) < C(q_i) \end{aligned}$$

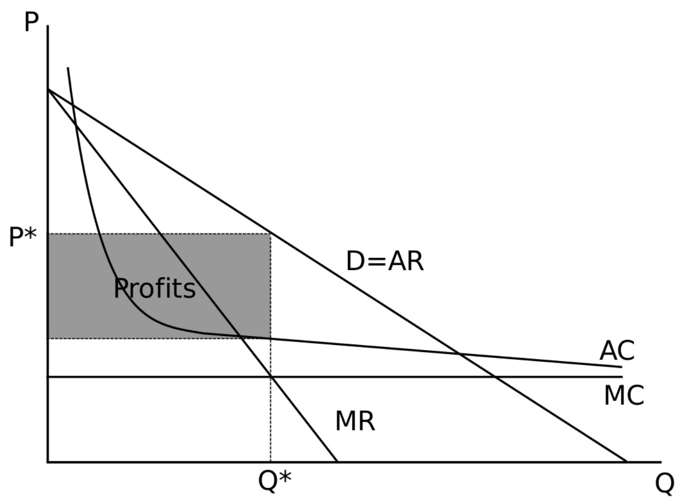
where $R(q_i, p_i)$ is the revenue of producing q_i at a price q_i. But if the profit of producing any quantity q_i under perfect competition is negative, then no firm would choose to enter and operate in that market. This essentially shows the general incompatibility of perfect competition and technologies with increasing returns to scale. For that reason, we assume that a firm producing variety i, displays some market power in the sense that it understands that changing its price implies a change in the demand function through (2). That is, the firm producing variety i behaves as a monopolist. Such market structure is known in the literature as *monopolistic competition*[4].

As usual the optimal decision for a firm implies that the marginal revenue equates the marginal cost:

$$MR(q_i, p_i) = MC(q_i)$$

which can be depicted in the traditional monopolist diagram in figure 1.

Figure 1: Optimal choice and profits of a monopolist



Note that in the graph, at the optimal choice for the monopolist output, profits given by the difference of the average revenue and the average cost are positive. In fact market power is fundamental to sustain an equilibrium in the case a firm has increasing returns to scale as, under perfect competition with $p = MC$, profits would be negative.

[4]Note from the demand equation (2) there are two impacts on consumption when a firm changes the price p_i: one direct (through the numerator of the term in brackets); another indirect (through the price index P in the denominator of the term in brackets). In a monopolistic competition market structure, we assume that the firm is only aware of the direct effect on demand. Therefore, we say that a monopolist producing i still faces competition from other firms that are producing close substitutes $j \neq i$ (which operates through a lower P). This assumption can be rationalized by saying that there are many varieties in the market, implying a small equilibrium firm size and a low influence in the general price level P. Instead, a pure monopolist would be aware of both the direct and the indirect effect of a price change in demand.

The same choice for the firm can be analyzed through a profit maximization problem. In particular, a i variety firm's profit is given by:

$$\begin{aligned} \pi_i &= q_i p_i - w l_i & (8) \\ &= q_i p_i - w\left(f + q_i/\varphi\right) \\ &= L c_i p_i - w\left(f + L c_i/\varphi\right) \\ &= L\left(\frac{p_i}{P}\right)^{-\sigma} C p_i - w\left(f + L\left(\frac{p_i}{P}\right)^{-\sigma} C/\varphi\right) & (9) \end{aligned}$$

where we made use of the fact that $q_i = Lc_i$, that is, a firm producing variety i has to supply all L consumers, each demanding c_i given by (2). Maximizing profits with respect to p_i, implies taking the derivative of (9) with respect to p_i and equating the equation to zero:

$$\begin{aligned} &\frac{\partial \pi_i}{\partial p_i}{=}0 \\ \Rightarrow \quad & 0{=}\left(1-\sigma\right) + \sigma w p_i^{-1}/\varphi \\ \Rightarrow \quad & p_i{=}\underbrace{\frac{\sigma}{\sigma-1}}_{markup} \cdot \underbrace{\frac{w}{\varphi}}_{marginal\,cost} \qquad \forall i = 1, ..., N & (10) \end{aligned}$$

This is the general price rule of a monopolist that faces a CES demand function as in (2). It is important to interpret this equation. We should note that the second term in the RHS is the marginal cost for the firm. The second term, which is always greater than one $\sigma/(1-\sigma) > 1$, implies that the firm overcharges a markup over the marginal cost. As we make σ larger, which implies that it is easier for the consumer to substitute one variety by another[5], the markup squeezes as $\sigma/(1-\sigma)$ becomes closer to 1.

Free entry An assumption that we are making in this model is that each firm produces a single variety. Thus, we need an additional condition to determine how many firms, and thus varieties, will emerge in equilibrium. A typical condition made in this kind of models is one related with *free market entry*. Note from figure 1 that a firm supplying a variety may operate with positive profits. It is thus natural to assume that, when incumbent firms are making positive profits, new firms will want to enter the market if they are allow to freely enter the market. That additional entry creates pressures on incumbent firms through

[5] Another interpretation for a larger σ is the demand curve is more elastic.

higher competition[6]. Such additional competition will eventual lead to lower profits to all firms. This process will continue until profits for all firms operate with *zero profits*.

Mathematically, zero profits imply that revenues equal costs. From equation (8), this means that:

$$
\begin{aligned}
\pi_i =& q_i p_i - w l_i = 0 \\
\Rightarrow \pi_i =& q_i p_i - w \left(f + \frac{q_i}{\varphi} \right) = 0 \\
\Rightarrow \frac{p_i}{w} =& \frac{f}{q_i} + \frac{1}{\varphi} \quad \forall i = 1, ..., N
\end{aligned} \tag{11}
$$

That is, under free entry, which implies zero profits, new firms will enter the market until the price of a variety in terms of labor p_i/w equals the RHS of (11)[7].

Market clearing conditions As usual, our general equilibrium is closed by imposing market clearing conditions. Because we have two main markets - market for varieties and market for labor - we need the following set of expressions that essentially equate supply and demand:

$$
\begin{aligned}
q_i &= Lc_i && \text{(varieties markets)} \\
L &= \sum_{i=1}^{N} l_i = Nf + \sum_{i=1}^{N} q_i/\varphi && \text{(labor market)}
\end{aligned}
$$

The equilibrium is thus given by the set of equations (labor market), (varieties markets), (11), (10), and (2).

A symmetric equilibrium Note that the solution to the set of all the equations summarized in (labor market), (varieties markets), (11), (10), and (2), can be potentially very cumbersome. An alternative way to characterize the equilibrium is to guess a particular solution and verify if such solution is consistent with the equilibrium conditions. A natural guess for a equilibrium is a symmetric one:

$$
p_i = p; \quad q_i = q; \quad c_i = c \quad \forall i = 1, ..., N
$$

[6]Mathematically speaking, when more firms enter the market, P falls in the demand equation $c_i = \left(\frac{p_i}{P}\right)^{-\sigma} C$, which implies that the demand curve shifts to the left, also implying lower profits.

[7]On its turn, because $\frac{p_i}{w} = \frac{\sigma}{\sigma-1}\frac{1}{\varphi}$, $q_i = f\varphi(\sigma - 1)$ as we will see in the next sections.

that is, all indexes drop from our previous equations. The reason for choosing this particular equilibrium derives from the fact that all firms are homogenous in technology (they share the same φ and f), and consumers value each variety equally (the coefficient associated with each $c_i^{\frac{\sigma-1}{\sigma}}$ in [1] is 1). We should now verify what are the implications of such equilibrium.

First note form (10) that the real price in terms of labor of each variety is constant and equal to:

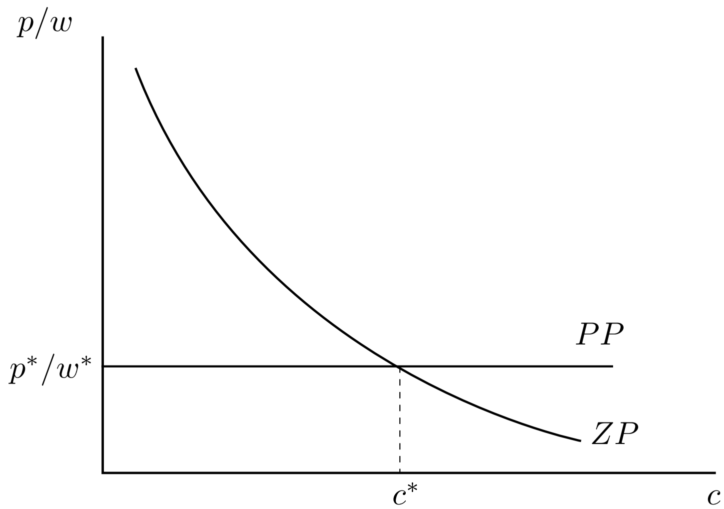
$$\frac{p}{w} = \frac{\sigma}{\sigma-1} \cdot \frac{1}{\varphi} \tag{PP}$$

Also note from (11) that the real price in terms of labor can be also be characterized by:

$$\frac{p}{w} = \frac{f}{q} + \frac{1}{\varphi} = \frac{f}{cL} + \frac{1}{\varphi} \tag{ZP}$$

where p/w is increasing in c. Equation (PP) is known as the optimal profits equation while (ZP) is the zero profit equation. As usual, the intersection between the two give us the equilibrium as shown in figure .2

Figure 2: Autarky equilibrium in the Krugman model



Comparative statics exercises can easily be done with recourse to the diagram. Consider, for example, that the fixed cost suddenly increase. Intuitively, we should realize that because now the fixed cost is large, firms need to generate higher profits to sustain the higher cost of f. However, larger profits can only be achieved if some firms exit the market, thus increasing the quantity produced and consumed per variety, q and c, respectively. The new equilibrium can be achieved by shifting the (ZP) to the right. Similar exercises can be done for shifts in L, φ, and σ.

Equating (PP) and (ZP) give us the equilibrium level of prices and consumption per variety:

$$\frac{p}{w} = \frac{\sigma}{\sigma-1}\cdot\frac{1}{\varphi} \tag{12}$$

$$c = \frac{f}{L}\varphi\left(\sigma-1\right) \tag{13}$$

Number of varieties and firms under a symmetric equilibrium To determine the number of firms that will be operating in equilibrium in this economy, we make use of the market clearing condition (labor market) and the equilibrium consumption per variety (13):

$$L{=}Nf + \sum_{i=1}^{N} q_i/\varphi$$

$$\Rightarrow L{=}Nf + Nq/\varphi$$

$$\Rightarrow N{=}\frac{L}{f+q/\varphi}$$

$$\Rightarrow N{=}\frac{L}{f+Lc/\varphi}$$

$$\Rightarrow N{=}\frac{L}{f+L\frac{f}{L}\varphi\left(\sigma-1\right)/\varphi}$$

$$\Rightarrow N{=}\frac{L}{f\sigma} \tag{14}$$

Price index and utility Note that the equilibrium is completely characterized with (12), (13), and (14). Then we should be able to compute what is the welfare that this equilibrium generates to consumers. In particular, we should be able to say if a particular shock to this economy improves the consumers' wellbeing.

A particular simple way of studying that the welfare consequences of the equilibrium is to look at the price index P. Recall that we saw that expenditure of a consumer equals its income (5):

$$\sum_{i=1}^{N} p_i c_i \equiv E = PC = w$$

where we are defining $E \equiv \sum_{i=1}^{N} p_i c_i$. Normalizing the expenditure on varieties in terms of labor yields:

$$\frac{E}{w} = \frac{P}{w} \cdot C$$

Note here that C is the level of utility from consuming all varieties. Thus, if we normalize the level of expenditure to 1, $E/w = 1$, we can describe utility in terms of the price index:

$$C|_{E/w=1} = 1/(P/w) \tag{15}$$

Thus a simple way to understand what are the implications of the model in terms of welfare is to look a the price index: if the price index increases the consumer is worse off, if decreases the consumer is better off[8].

From the definition of P in (3), the price index can be represented as:

$$\begin{aligned} P &= \left(\sum_{i=1}^{N} p_i^{1-\sigma} \right)^{\frac{1}{1-\sigma}} \\ &= N^{\frac{1}{1-\sigma}} p \end{aligned} \tag{16}$$

The previous equation immediately tells us that prices decrease for two potential reasons: either because the number of varieties N increase, or because the individual price of a variety p falls. Substituting the equilibrium conditions in (16) yields:

$$P = \left(\frac{L}{f\sigma} \right)^{\frac{1}{1=\sigma}} \frac{\sigma}{\sigma - 1} \cdot \frac{w}{\varphi}$$

Substituting this expression in our utility representation in (15) gives:

$$C|_{E/w=1} = \left(\frac{L}{f\sigma} \right)^{\frac{1}{\sigma-1}} \frac{\sigma - 1}{\sigma} \cdot \varphi \tag{17}$$

This last expression can also be verified directly using the utility function (1) and the

[8]Recall also that P can be interpreted as the expenditure required to achieve one unit of the aggregate utility. Thus a lower P implies that one unit of aggregate utility becomes cheaper.

equilibrium conditions for c and N, (13) and (14), respectively:

$$\begin{aligned}
C &= \left(\sum_{i=1}^{N} c_i^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} \\
&= N^{\frac{\sigma}{\sigma=1}} c \\
&= \left(\frac{L}{f\sigma}\right)^{\frac{\sigma}{\sigma=1}} \frac{f}{L}\varphi\left(\sigma-1\right) \\
&= \left(\frac{L}{f\sigma}\right)^{\frac{\sigma}{\sigma=1}} \frac{f\sigma}{L}\varphi\frac{(\sigma-1)}{\sigma} \\
&= \left(\frac{L}{f\sigma}\right)^{\frac{\sigma}{\sigma=1}-1} \varphi\frac{(\sigma-1)}{\sigma} \\
&= = \left(\frac{L}{f\sigma}\right)^{\frac{1}{\sigma=1}} \varphi\frac{(\sigma-1)}{\sigma}
\end{aligned}$$

which is consistent with (17). Using this formula it is easier to understand the impact on welfare from shocks to the economy. For example an increase in f, should reduce the number of firms that operate in the market (thus decreasing the number of varieties), while maintaining the price per variety constant. Thus the price index for the economy increases, implying that the consumers become worse off.

One of the advantages of working with the Krugman model is the simplicity of its exposition. Adding a similar trading partner to this economy becomes almost a trivial exercise. That is done in the next section.

1.2 The Krugman's model with free trade between two identical economies

Recall from the classical theories of trade that exchange between economies was always based on the comparative advantage. In particular a country would trade with another if its autarky relative price differs from the trading partner. This could only be the case if the countries were different around any particular dimension (productivity, endowments, factor proportions, preferences, etc.). We will see that under the Krugman model, trade emerges even for economies that are similar in every dimension. Two reasons explain why that is the case: consumers in the home economy benefit from having access to the foreign economy set of varieties; and firms can expand their scale of production thus lowering their average costs (due to the fact of increasing returns to scale).

To see what happens when we allow for trade between two identical economies (home H, and foreign F), lets start by noting that the (PP) equation doesn't change:

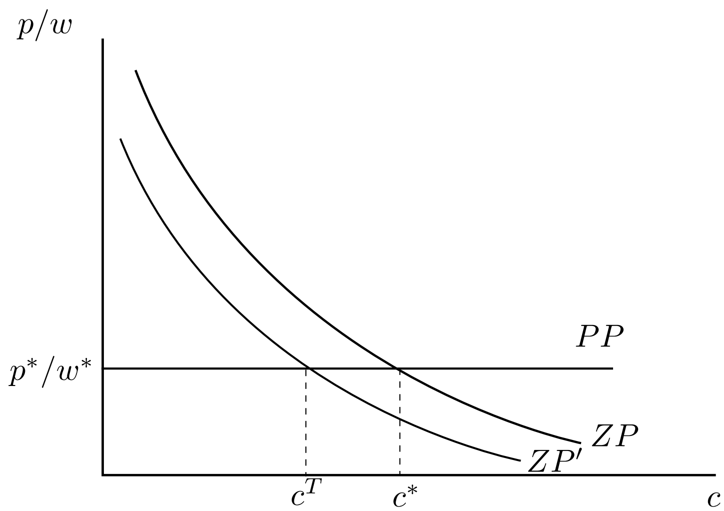
$$\frac{p}{w} = \frac{\sigma}{\sigma - 1} \cdot \frac{1}{\varphi} \quad \text{(PP-FT)}$$

That is because firms in both economies still face the same type of demand and the same technology. At the same time, the zero profit equation changes to accommodate the fact that the integrated world market has now $2L$ consumers. Thus, market clearing conditions imply $q_i = 2Lc_i$, that is each home firm has to supply the demand of both domestic and foreign consumers. This implies that (ZP) becomes:

$$\frac{p}{w} = \frac{f}{q} + \frac{1}{\varphi} = \frac{f}{c2L} + \frac{1}{\varphi} \quad \text{(ZP-FT)}$$

But note that this is just a shift to the left side of the (ZP) equation as shown in figure 3.

Figure 3: Free-trade equilibrium in the Krugman model



It is obvious from the figure that under trade, the domestic individual consumption per variety falls. In particular, we can equate (PP-FT) and (ZP-FT) to get:

$$c^{FT} = \frac{f}{2L}\varphi(\sigma - 1) = \frac{1}{2}c^A < c^A$$

where c^{FT} is the consumption under free trade, while c^A is the consumption under autarky given by (13). Despite the fact that individual consumption falls to 1/2, the quantity produced by a domestic firm stays unaltered to:

$$q^{FT} = f\varphi\left(\sigma - 1\right) = q^{A}$$

This is because firms now have access to, not just the domestic consumers, but also to foreign consumers. Essentially, because prices don't change, implying that firms also don't have any incentive to change the scale of production. In a broader sense, firms face are two effects in an open economy: on one hand they have access to a wider which would imply a lower average cost and thus higher profits; on the other hand, there's a increased competition effect where domestic firms' varieties need now to compete with foreign firms' varieties, thus pushing profits down. Turns out that with a CES utility function of the form in (1), these two effects cancel each other and, for that reason, firms don't change their optimal scale of production. We will see further that this is specific to the utility function. In general, the (PP) can be upward slopping, which would imply that prices would change and thus firms would also increase their scale of production.

Given the equilibrium quantities and prices, we can now turn gears to the remaining variables that characterize the economy. In particular note that the number of varieties produced domestically doesn't change under free trade:

$$\begin{aligned} L &= Nf + Nq/\varphi \\ \Rightarrow N^{FT} &= \frac{L}{f\sigma} = N^{A} \end{aligned}$$

Note however that, despite the fact that the number of domestically produced varieties doesn't change, consumers have now access to both domestic and foreign produced varieties. This should have positive consequences for welfare as consumers love additional varieties:

$$P^{FT} = \left(2N^{FT}\right)^{\frac{1}{1-\sigma}} p = 2^{\frac{1}{1-\sigma}} P^{A} < P^{A}$$

Also, using (15), we can see that individual utility becomes:

$$C^{FT} = \left(2N\right)^{\frac{1}{\sigma-1}} \varphi \frac{\sigma - 1}{\sigma} = 2^{\frac{1}{\sigma-1}} C^{A} > C^{A}$$

Which essentially confirms our intuition that consumers are better off under free trade.

Recap of the Krugman model under free-trade (and CES utility) To understand what is going on on the free-trade equilibrium of the Krugman model, we should understand that the only source here of gain from trade is the fact that consumers have now access to a wider variety range. Because prices can't change (due to the CES utility assumption), firms don't change their output level per variety. This implies that now each firm is selling half of it's production to the domestic market, and another halt to the foreign market. Recall that both economies produce varieties in the same sector. But then the trade observed between the two countries is purely intra-industrial domestic firms export the same industry goods that domestic consumer demand in the form of imports. We should stress that the access to more varieties is not the only potential source of gains from trade in a Krugman type of economy. Under a general assumption for the utility function, firms can also increase their scale of production as a result of shifting from autarky to free trade, thus benefiting from the increasing returns to scale.

On the negative side of this model, we have the result that all firms behave identically due to the symmetric equilibrium. We know from the data that this is a highly counterfactual implication as firms display stronger heterogeneity that is also verified along the exporting/non-exporting dimension.

Finally we should stress that the Krugman model is simple and quite tractable. For that reason several extensions can be accommodated within its framework to make its key results more realistic.

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