Practice Assignment 1 and Solution Topics

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1. **Model of Exchange Trade**
   The home economy an endowment of $e_1 = 1$, $e_2 = 1$ and a utility function given by:

$$u(x_1, x_2) = \alpha \log x_1 + (1 - \alpha) \log x_2$$

   While the foreign economy has $e_1^* = 3$, $e_2^* = 1$ and

$$u^*(x_1^*, x_2^*) = \alpha^* \log x_1 + (1 - \alpha^*) \log x_2^*$$

   (a) Solve for the terms of trade when we allow for free trade between the domestic and the foreign economy.
   From both consumers maximization problems we have the following demand equations for $p = p_1/p_2$:

$$\begin{aligned} x_2 &= (1-\alpha)(pe_1 + e_2) \\ x_1 &= \alpha(e_1 + e_2/p) \\ x_2^* &= (1-\alpha^*)(pe_1^* + e_2^*) \\ x_1^* &= \alpha^*(e_1^* + e_2^*/p) \end{aligned}$$

   which together with market equilibrium conditions:

$$\begin{aligned} x_1 + x_1^* &= e_1 + e_1^* \\ x_2 + x_2^* &= e_2 + e_2^* \end{aligned}$$

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Implying the following price ratio:

$$p = \frac{\alpha e_2 + \alpha^* e_2^*}{(1-\alpha)\, e_1 + (1-\alpha^*)\, e_1^*} \tag{1}$$

Thus substitution of (1) on the demand equations gives the equilibrium consumptions in this world economy.

(b) Show that at equilibrium prices, the choice for each economy to consume its own endowment is feasible.

Consuming its own endowment corresponds to $x_i^j = e_i^j$. It is clear that for any price p that allocation is feasible (but not necessarily optimal).

(c) What happens to the terms of trade when $e_1 = 2$? And when α^* increases? Very succinctly does these changes make sense?

Direct inspection of (1) shows that more e_1 implies a fall in p (more world abundance of endowment 1 implies a fall in the equilibrium price). Similarly an increase in α^* implies an increase in p (stronger demand for good 1 implies an increase in price).

2. **A trade model with specialization**

Consider now an economy similar to the previous one but where instead of endowment for each good, we have output that uses labor in the production process:

$$\begin{aligned} y_1 &= z_1 l_1 \\ y_2 &= z_2 l_2 \\ y_1^* &= z_1^* l_1^* \\ y_2^* &= z_2^* l_2^* \end{aligned}$$

Let $(z_1, z_2, z_1^*, z_2^*) = (1, 1, 3, 5)$, and the total labor available in the domestic economy be L and L^* for the foreign economy.

(a) In the handout, we showed that under autarky, the first order condition for x_1 is:

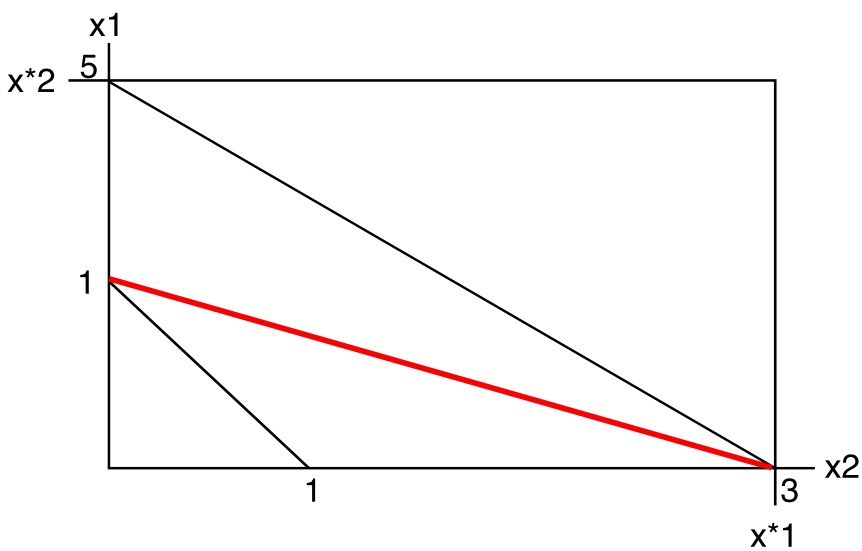
$$\alpha \frac{1}{x_1} = \lambda p_1$$

where λ is a Lagrange multiplier. Show that $\lambda = 1/(wL)$.

This can be easily seen by substituting the first order conditions in the budget constraint:

$$\begin{aligned} x_1p_1 + x_2p_2 &= wL \\ \Rightarrow \frac{\alpha}{\lambda} + \frac{1-\alpha}{\lambda} &= wL \\ \Rightarrow \qquad \lambda &= \frac{1}{wL} \end{aligned}$$

(b) Draw in a y_1, y_2 graph the production possibility frontier for each country individually and for the entire world. Briefly, what can we conclude?
To simplify our analysis let $L = L^* = 10$. One possible graph can the following:



Note here that the world as a whole can trade in goods with each other, that mean that for a particular terms of trade, observing that:

$$\frac{1/z_1}{1/z_2} = \frac{1}{1} \leq p \leq \frac{1/z_1^*}{1/z_2^*} = \frac{5}{3}$$

For such prices one can see that there both countries can expand consumption beyond the initial production possibility frontier.

(c) Let $\alpha = \alpha^* = 0.3$, $L = 10$, and $L^* = 10$. Solve for the free-trade general equilibrium. Is this equilibrium characterized by full specialization? And when $L = 20$?

Let's first check for the full specialization general equilibrium. Under that scenario, prices would be given by:

$$p = \frac{\alpha}{1-\alpha} \cdot \frac{z_2^*}{z_1} = \frac{15}{7}$$

which would be larger than the autarky prices for the foreign economy:

$$p = \frac{1/z_1^*}{1/z_2^*} = \frac{5}{3}$$

Thus, prices under free trade will reflect some partial specialization, that is, they should be bounded to:

$$p = \frac{5}{3}$$

Under this price ratio, allocations of consumption are given by the following:

$$\begin{aligned}
x_1^* &= \alpha z_1^* L^* = 9 \\
x_2^* &= (1-\alpha)\, z_2^* L^* = 35 \\
x_1 &= \alpha z_1 L = 3 \\
x_2 &= (1-\alpha)\, \frac{z_2^*}{z_{1^*}} z_1 L = 35/3
\end{aligned}$$

While labor allocations are:

$$\begin{aligned}
l_1 &= L = 10 \\
l_2 &= 0 \\
l_1^* &= \alpha L^* - (1-\alpha)\, \frac{z_1}{z_1^*} L = 3 - 7/3 \\
l_2^* &= (1-\alpha)\, L^* + (1-\alpha)\, \frac{z_1}{z_{1^*}} L = 7 + 7/3
\end{aligned}$$

3. Very briefly (you can just give some general ideas), where do you think the Ricardian model is unrealistic.

   Examples of topics worth discussion are: single factor of production (constant opportunity cost); full specialization (at least in one country) when reality may not support this; workers never loose from free trade (but why are unions against free trade then?)