Practice Assignment 2 and Solution Topics

Tiago Tavares*
International Trade I at ITAM

1. **Ricardian model with a continuous of goods**
   Let's depart from the Ricardian model extension with a continuous of goods that we already studied. Assume that the relative labor requirements between the domestic and foreign economy has the following form:

$$A(i) = \frac{1/z(i)}{1/z^*(i)} = \sqrt{i}, \ \ i \in [0,1]$$

   while the preferences of both economies are characterized by a uniform distribution:

$$\int_0^1 b(i)di = \int_0^1 bdi$$

   (a) Prove that there exist a $\hat{i}$ such that the domestic economy produces all goods such that $i < \hat{i}$ and the foreign economy all goods $i > \hat{i}$.
   Suppose there exists a $i' < i$ such that i is produced at home and i' is produced at foreign. Then:

$$\begin{aligned} p\,(i)\,z(i) &= w \\ p(i')z(i') &\leq w \\ p(i)z^*(i) &\leq w^* \\ p(i')z^*(i') &= w^* \end{aligned}$$

   But then

$$p(i')z(i')p(i)z^*(i) \leq w \cdot w^* = p\,(i)\,z(i)p(i')z^*(i')$$

*Email: Please contact me to tgstavares@gmail.com if you find any errors in the text.

This implies

$$A(i) \equiv \frac{z^*(i)}{z(i)} \leq \frac{z^*(i')}{z(i')} \equiv A(i')$$

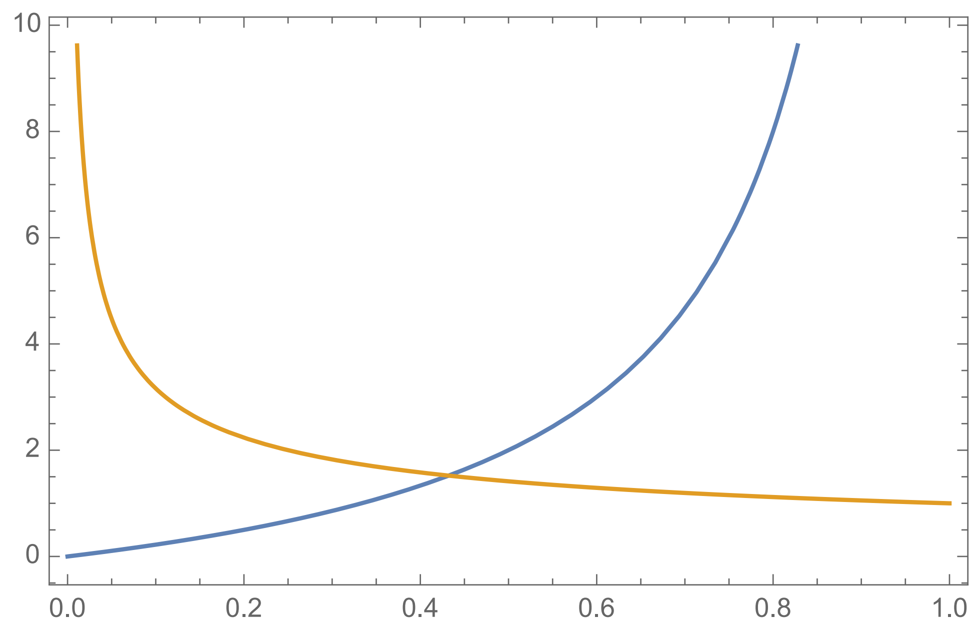
And this contradicts the fact that $A(i)$ is increasing.

(b) Show that $b = 1$. Find a formula for $\theta(i) = \int_0^i b(i)di$, the expenditure share on domestically produced goods

Note that $\int_0^1 bdi = b\, i|_0^1 = b$, so it must that $b = 1$ in order for the integral equal 1. Substituting $\theta(i) = \int_0^i b(i)di = \int_0^i di = i$

(c) Compute the equilibrium $\hat{i}$ and w/w^* assuming that $L = 10$ and $L^* = 20$.

The equilibrium is given by the intersection of $A(i)$ with $B(i) = \frac{i}{1-i} \cdot \frac{L^*}{L}$ as in the next figure

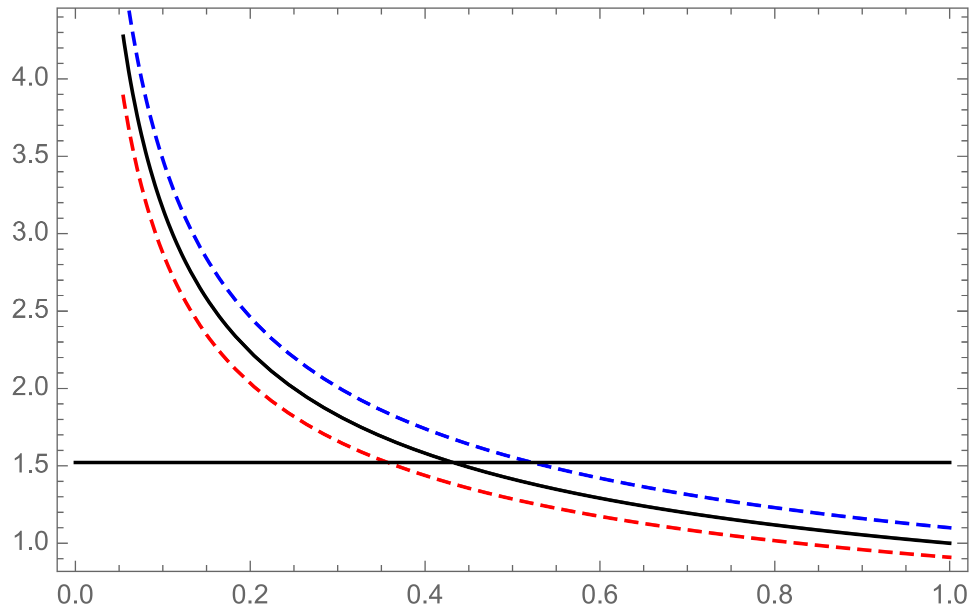


Using numerical methods, the solution is given by:=

$$\begin{aligned} \hat{i} &= 0.43 \\ w/w^* &= 1.52 \end{aligned}$$

(d) For the relative wage you calculated in the previous question, what is the share of goods that is not traded if we impose a transportation cost of 10%

Now the solution is represented in the following picture:



That is we use two curves:

$$A^u(i) \;=\; \sqrt{i}\cdot 1.1$$
$$A^l(i) \;=\; \sqrt{i}/1.1$$

and calculate the intersection with $w/w^* = 1.52$, yielding $i^l = 0.36$ and $i^u = 0.52$. So the share not traded is $\Delta = 0.52 - 0.36 = 0.16$

(e) Please give some arguments on the welfare impact when L^* increases (no need for calculations)

Note that as L^* increases, w/w^* increases too (at initial wages, an increase in L^* creates a trade deficit abroad that is closed with a reduction of w^*); part of the production will be shift to the foreign country; for goods i that continue to be produced at home, nothing change in $p(i)$; but for goods that move production to abroad, $p(i)$ falls. Overrall, the domestic economy finds an improvement in the terms of trade thus benefiting welfare. Similar arguments show that the foreign economy observe a fall in welfare.

(f) Suppose that the foreign country decides to transfer an amount T of its income to the home country. What are the implications for trade, ie, how does production in this world economy changes?

Now the home's country income becomes $wL + T$ while the foreign countries income $w^*L^* - T$. Because of the transfer, the home economy can now import more that what it exports in the amount of T. Hence the new relevant market clearing condition becomes:

$$imports - exports = T \Rightarrow \left(1 - \theta(\hat{i})\right)(wL + T) - \theta(\hat{i})(w^*L^* - T) = T$$

Which is equivalent to

$$\left(1 - \theta(\hat{i})\right) wL = \theta(\hat{i}) w^* L^*$$

Hence, neither patterns of trade, nor terms of trade would change with this transfer.

2. **Specific factors model**
In the sector-specific model, suppose that the domestic and foreign economies have identical labor and capital in sector 2, but domestic has more capital in sector 1 (technologies and preferences are the same across countries).

   (a) Can we predict the trade pattern between countries? Which factors at home benefit from the opening of trade and what factors loose?
   Because home has more capital in sector 1, more output will be produced of good 1, impling a lower price of that good. This implies that the home economy would have a comparative advange in the production of 1. But then, opening up to trade would lead to an increase in the price of 1. This inequivocally increases the real return of the factor specific to sector one, while reducing the factor real return in the other sector. Real wages would have an ambiguous change.

   (b) Now suppose that both home and foreign has the same capital across sectors, but home has more labor. Without any math derivation (you can use diagrams), can we predict the pattern of trade (try to use assumptions on the factor cost shares to help determine this)?
   Without an assumption on the relative intensity utilization of labor in each industry, we cannot make a prediction. But suppose industry 1 is more intensive in labor. Then, abundance in labor implis a comparative advantage in sector 1. Thus opening up to trade would increase the relative price of good 1, implying an increase in the real return of factor 1 against a fall in the real return of factor 2. Again, we would have an ambiguous effect on real wages.

3. **Heckscher-Ohlin model**

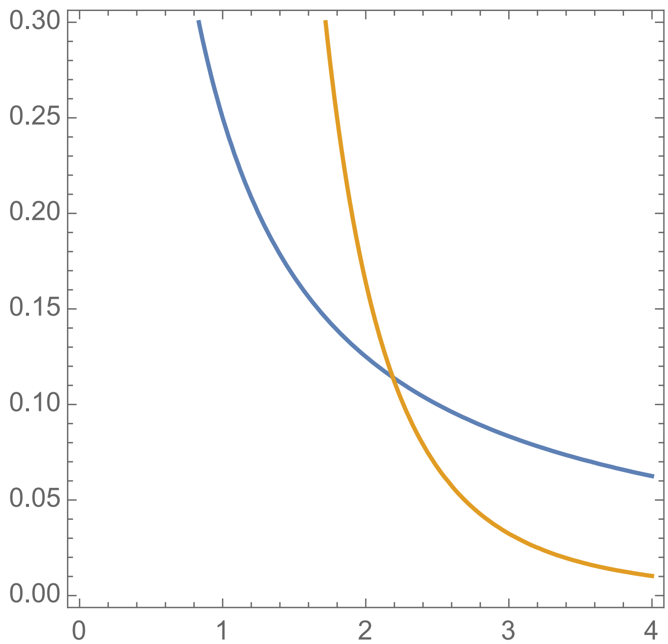
Let an economy be characterized by the following production functions:

$$\begin{aligned} y_1 &= f^1(K_1, L_1) = K_1^{0.5} L_1^{0.5} \\ y_2 &= f^2(K_2, L_2) = K_2^{0.2} L_2^{0.8} \end{aligned}$$

and $K = 10$; $L = 1.5$; Assume a small open economy with $p_1 = 1$ and $p_2 = 2$;

(a) Solve for each sector firms cost minimization problem for one unit of production and check that the factor intensity reversal never holds in this economy.

That factor intensity reversal never hold can be seen from solving the firms cost minimization problem:



Implying the following demand functions:

$$\begin{aligned} k_i\left(w, r\right) &= \left(\frac{\alpha_i w}{(1-\alpha_i 1)r}\right)^{1-\alpha_i} \\ l_i\left(w, r\right) &= \left(\frac{(1-\alpha_i)r}{\alpha_i w}\right)^{\alpha_i} \end{aligned}$$

for $i = 1, 2$

(b) At these final good prices, what are the equilibrium factor prices and the relative

labor intensity in both sectors? What can you predict that happens to the factor prices when p_1 increases? Calculate the new factor prices when $p_1 = 1.1$. Is this consistent with the Stolper-Samuelson theorem?
The equilibrium factor prices are then given by:

$$w = 2.19; \;\; r = 0.11$$

with

$$\begin{aligned} l_1/k_1 &= 0.052 \\ l_2/k_2 &= 0.21 \end{aligned}$$

With $p_1 = 1.1$ we now have

$$w = 2.05; \;\; r = 0.15$$

with

$$\begin{aligned} l_1/k_1 &= 0.071 \\ l_2/k_2 &= 0.29 \end{aligned}$$

These results fit the Stolper-Samuelson theorem predictions.

(c) Calculate the output vector of this economy $(y1, y2)$.
That is given by:

$$y1 = 0.857083, \;\; y2 = 1.78413$$