Mock Midterm - Suggested Solution

International Trade I at ITAM

October 2015

1. Consider a world with 2 countries home (h) and foreign (f). Both countries produce final goods x and y using the following technologies ($i = h, f$):

$$\begin{aligned} x_i &= k_{ix}^{\alpha} l_{ix}^{1-\alpha} \\ y_i &= k_{iy}^{\beta} l_{iy}^{1-\beta} \end{aligned}$$

where $\beta > 0$, $\alpha > 0$, $\beta > \alpha$ and k, l are factors of production. Assume that these factors of production are perfectly mobile across sectors.

   (a) Under autarky, set up firms maximization problem and obtain the first order conditions for countries $i = h, f$.

   Firms maximization problem is characterized by the solution of the following problem:

$$\max_{x_i} \{p_{xi}x_i - w_i l_{xi} - r_i k_{xi}\}$$
$$st$$
$$x_i = k_{ix}^{\alpha} l_{ix}^{1-\alpha}$$

   Where first order conditions are given by:

$$\begin{aligned} p_{xi}\alpha k_{ix}^{\alpha-1} l_{ix}^{1-\alpha} &= r_i \\ p_{xi}(1-\alpha) k_{ix}^{\alpha} l_{ix}^{-\alpha} &= w_i \end{aligned}$$

Proceeding in the same fashion we also have

$$\begin{aligned} p_{yi}\beta k_{iy}^{\beta-1}l_{iy}^{1-\beta} &= r_i \\ p_{yi}(1-\beta)k_{iy}^{\beta}l_{iy}^{-\beta} &= w_i \end{aligned}$$

from both $i = h, f$.

(b) Using the first order conditions, express the capital-labor ratio of each sector as a function of the ratio of the wage rate and rental rate of capital (w_i/r_i).
Using the equations we just derived one gets:

$$\begin{aligned} \frac{\alpha}{(1-\alpha)}\frac{w_i}{r_i} &= \frac{k_{ix}}{l_{ix}} \\ \frac{\beta}{(1-\beta)}\frac{w_i}{r_i} &= \frac{k_{iy}}{l_{iy}} \end{aligned}$$

(c) Under the same assumptions what are the equilibrium price ratios p_{xi}/p_{yi} as a function of the ratio of the wage rate and rental rate of capital?
To get the price ratio, one just needs to divide one foc into the other

$$\frac{p_{xi}\alpha\left(k_{ix}/l_{ix}\right)^{\alpha-1}}{p_{yi}\beta\left(k_{iy}/l_{iy}\right)^{\beta-1}} = 1$$

substituting with the previous expression and rearranging:

$$\frac{p_{xi}}{p_{yi}} = \frac{\beta^{\beta}(1-\beta)^{1-\beta}}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}}\left(\frac{w_i}{r_i}\right)^{\beta-\alpha}$$

(d) What forces explain the differences in comparative advantage between these two countries?
Note that the price ratio is a function of the wage rate relative to the rental rate of capital. It follows that if the home country is more abundant in labor relative to capital than the foreign country, then, it's w_h/r_h will be lower and, thus, p_{xh}/p_{yh} will also be lower (as $\beta > \alpha$, meaning that the production of x is more labor intensive than y). Note that this is the Heckscher-Ohlin force leading to differences in comparative advantage.

(e) Under free trade and when both countries produce both goods, will factor price equalization hold? Why or why not?

It will as a direct result of the factor price equalization theorem. Recall that under free trade $p_{xh} = p_{xf}$ and $p_{yh} = p_{yf}$. So the previous equation

$$\frac{p_x}{p_y} = \frac{\beta^\beta (1-\beta)^{1-\beta}}{\alpha^\alpha (1-\alpha)^{1-\alpha}} \left(\frac{w_i}{r_i}\right)^{\beta-\alpha}$$

Can only true for $i = h, f$ when $w_f = w_h$ and $r_f = r_h$.

2. A small country has been in total isolation, with labor input coefficients (labor is the only factor of production) shown by $a_{l1} = 10$, $a_{l2} = 8$, $a_{l3} = 12$, $a_{l4} = 7$.

   (a) In this autarky state, how many units of the first commodity would be required to exchange for a single unit of the third commodity?
   The amount of commodity 1 to be given up to obtain 1 unit of 3 is $a_{l3}/a_{l1} = 6/5$.

   (b) This country is now open to free trade with the rest of the world. Prices in the rest of the world are given by $p_1 = 1$, $p_2 = 8$, $p_3 = 10$, and $p_4 = 2$. What is the pattern of production in this small country after trade is opened up?
   The country will produce commodity 2 since its ratio of price to technical labor-hours required to produce one unit of it (p_i/a_{li}) is highest.

   (c) Show what happens to the real income of a worker in the small country who consumes only the fourth commodity before and after trade?
   The real income of the worker before trade is $1/a_{l4} = 1/7$. After trade, the real income is w/p_4 where w is equal to $p_2/a_{l2} = 1/2$ (since the country specializes in the production of commodity 2).

3. Within the specific-factor model environment, explain why Australian capitalists and landowners probably favor the same policy toward immigration. Given the traditional export position of Australian wool in world markets, how might owners of sheep stations (that use land) be expected to react to an increase in domestic prices of manufacturers (that use capital) brought by a tariff? Through what mechanism might land rents be distributed?
   If there is immigration, wages will fall. The competitive profit conditions then imply that the returns to both capital and land should increase. A tariff on manufactures will raise their price. This will raise the value of the marginal product of labor in the manufacturing sector, thus attracting workers away from sheep farming. The loss of

labor in the sheep sector will reduce the returns to land, thus hurting sheep station owners.

4. Consider a two country, two good world where consumers in both home (h) and foreign country (f) value the goods accordingly to:

$$
\begin{aligned}
U^h(c_{hh}, c_{fh}) &= (c_{hh})^\alpha (c_{fh})^{1-\alpha} \\
U^f(c_{hf}, c_{ff}) &= (c_{ff})^\beta (c_{hf})^{1-\beta}
\end{aligned}
$$

where $c_{i,j}$ is consumption in country i from a good produced in country i. Assume there are no transportation costs.

(a) Given prices $p_{i,j}$, write a formula for the price index in both home and foreign countries.

Note that the price index is just the optimal expenditure to get 1 unit of utility. It follows that in order to derive the price index one just need to get the expenditure function from this consumer. As always the demand function of a Cobb-Douglas utility function is characterized by a constant fraction of total expenditure:

$$
\begin{aligned}
x_{hh} &= \alpha \frac{E}{p_h} \\
x_{fh} &= (1-\alpha)\frac{E}{p_f}
\end{aligned}
$$

Thus, indirect utility is given by

$$
\begin{aligned}
U^h(p_{hh}, p_{fh}, E) &= \left(\alpha \frac{E}{p_h}\right)^\alpha \left((1-\alpha)\frac{E}{p_f}\right)^{1-\alpha} \\
&= \left(\frac{\alpha}{p_h}\right)^\alpha \left(\frac{1-\alpha}{p_f}\right)^{1-\alpha} E
\end{aligned}
$$

Implying that the expenditure function is

$$
E^h(p_{hh}, p_{fh}, U) = \left(\frac{p_h}{\alpha}\right)^\alpha \left(\frac{p_f}{1-\alpha}\right)^{1-\alpha} U
$$

Thus, the perfect price index is given by:

$$
E^h(p_{hh}, p_{fh}, 1) = P_h = \left(\frac{p_h}{\alpha}\right)^\alpha \left(\frac{p_f}{1-\alpha}\right)^{1-\alpha}
$$

And similarly for foreign economy:

$$P_f = \left(\frac{p_f}{\beta}\right)^{\beta} \left(\frac{p_h}{1-\beta}\right)^{1-\beta}$$

(b) How does terms of trade relate with the ratio of the price indexes between home and foreign economy (this ratio in economics is also referred as the real exchange rate)? Interpret that relationship.

The real exchange rate is given by the ratio:

$$\begin{aligned} RER \equiv \frac{P_h}{P_f} &= \frac{\left(\frac{p_h}{\alpha}\right)^{\alpha}\left(\frac{p_f}{1-\alpha}\right)^{1-\alpha}}{\left(\frac{p_f}{\beta}\right)^{\beta}\left(\frac{p_h}{1-\beta}\right)^{1-\beta}} \\ &= \frac{(p_h/p_f)^{\alpha}\, p_f}{(p_f/p_h)^{\beta}\, p_h} \cdot \frac{\left(\frac{1}{\alpha}\right)^{\alpha}\left(\frac{1}{1-\alpha}\right)^{1-\alpha}}{\left(\frac{1}{\beta}\right)^{\beta}\left(\frac{1}{1-\beta}\right)^{1-\beta}} \\ &= \frac{tt^{\alpha}}{tt^{-\beta}} tt^{-1} \cdot \frac{\left(\frac{1}{\alpha}\right)^{\alpha}\left(\frac{1}{1-\alpha}\right)^{1-\alpha}}{\left(\frac{1}{\beta}\right)^{\beta}\left(\frac{1}{1-\beta}\right)^{1-\beta}} \\ &= tt^{\alpha+\beta-1} \cdot \frac{\left(\frac{1}{\alpha}\right)^{\alpha}\left(\frac{1}{1-\alpha}\right)^{1-\alpha}}{\left(\frac{1}{\beta}\right)^{\beta}\left(\frac{1}{1-\beta}\right)^{1-\beta}} \end{aligned}$$

That is, the real exchange rate is non-linear function of the terms of trade. In particular, if there is home bias in consumption ($\beta > 0.5$ and $\alpha > 0.5$), the elasticity of the real exchange rate with respect to the terms of trade is positive, that is, they move together. If, however, there's no home bias ($\beta = \alpha = 0.5$), then the so called purchasing power parity holds in this economy, that is, goods across countries tend to have the same prices when converted to the same currency unit.