Midterm - Suggested Solution Topics

International Trade I at ITAM

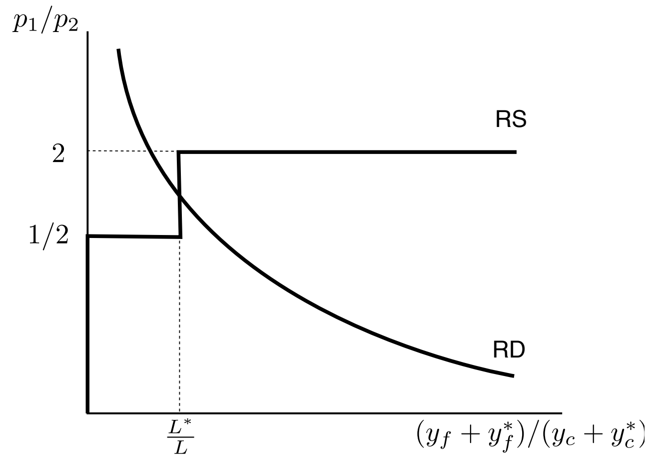
October 14 2015

1. (35) Lets consider a world of two economies (home and foreign) with identical and homothetic preferences. Suppose the production in the home country requires two labor-hours per unit of food and only one labor-hour per unit of clothing; whereas the foreign country requires one labor-hours per unit of food and only two labor-hour per unit of clothing. The foreign country's labor force consists of 1 million labor-hours.

   (a) If the home country is enough small relative to the foreign country, one of the countries will produce both goods. Which country? What will be food's relative price?
   Foods relative price would be determined by the largest country, in this case foreign's: $p_f/p_c = a^*_f/a^*_c = 1/2$.

   (b) If the home country is enough large relative to the foreign country, one of the countries will produce both goods. Which country? What will be food's relative price?
   A similar argument justifies that the price will be: $p_f/p_c = 2$

   (c) Construct the world demand and supply schedules for the relative quantity of food with respect with clothing.



(d) Suppose preferences are given by:

$$u(x_c, x_f) = 0.5 \log x_c + 0.5 \log x_f$$

where x_c, x_f are the consumption of clothing and food. Let home economy have also 1 million labor-hours labor force. What will be the equilibrium world prices in this economy? How would the labor force in home country need to change in order for us to observe home producing both goods?
Note that demand curves will be such that (similar to foreign):

$$\begin{aligned} p_f x_f &= 0.5wL \\ p_c x_c &= 0.5wL \end{aligned}$$

But then relative world demand is:

$$\frac{p_f}{p_c} = \frac{1}{\left(x_f + x_f^*\right) / \left(x_c + x_c^*\right)}$$

Equating this with supply gives an equilibrium price of $p_f/p_c = 1$
Note that a price of 2 would have imply:

$$2 = \frac{1}{\left(x_f + x_f^*\right) / \left(x_c + x_c^*\right)} \Rightarrow \left(x_f + x_f^*\right) / \left(x_c + x_c^*\right) = 1/2$$

So, because the foreign country has 1M workers, home country would have need to have 2M or more workers to sustain such price.

(e) Show that the welfare increases for the home economy when we shift from autarky to free trade.

Note the budget equals:

$$\frac{p_f}{p_h}x_f + x_c = \frac{w}{p_h}L$$

But p_f/p_h falls and w/p_h increases (as home country becomes an exporter of clothing). Thus the budget constraint expands, implying higher welfare after opening up to trade.

2. (20) Suppose only one technique can be used in the production of clothing. To produce a unit of clothing requires four labor-hours and one unit of capital; in food production each unit requires a single labor-hour and a unit of capital. At an initial equilibrium suppose the wage rate and the capital rental rate are each valued at $2. If both goods are produced, what must be their prices? Now, keep the price of food constant and raise the price of clothing to $15.

Note that

$$\begin{aligned} p_c &= 4 \cdot 2 + 2 = 10 \\ p_f &= 2 + 2 = 4 \end{aligned}$$

(a) State, in words, the Samuelson-Stolper theorem.

An increase in the relative price of a good will increase the real return to the factor used intensively in that good, and reduce the real return to the other factor

(b) Trace through the effect on the distribution of income.

With $p_c = 15$ we have:

$$\begin{aligned} 15 &= 4 \cdot w + r \\ 4 &= w + r \end{aligned}$$

Thus, $w' = 3.66$ and $r' = 0.33$ from $w = 2$ and $r = 2$; while $p'_c = 15$ and $p'_f = 4$ from $p'_c = 10$ and $p'_f = 4$

(c) Rank the relative changes in the wage rate, the price of clothing and the price food, and the rental rate of capital.

Using the typical notation:

$$\dot{w} > \dot{p}_c > \dot{p}_f > \dot{r}$$

3. (30) Consider a world with 2 countries home (h) and foreign (f). Both countries produce final goods x and y using the following technologies ($i = h, f$):

$$\begin{aligned} x_i &= z_{ix} k_{ix}^{\alpha} l_{ix}^{1-\alpha} \\ y_i &= z_{iy} k_{iy}^{\beta} l_{iy}^{1-\beta} \end{aligned}$$

where $\beta > 0$, $\alpha > 0$, $\beta > \alpha$ and k, l are factors of production. The parameters z_{ix} and z_{if} are the total factor productivity across sectors and across countries (we assume they can differ across countries). Assume that these factors of production are perfectly mobile across sectors.

(a) Under autarky, set up firms maximization problem and obtain the first order conditions for countries $i = h, f$.

$$\max_{k_{ix}, l_{ix}} \left\{ p_{x_i} x_i - w_i l_i - r_i k_{x_i} \quad st \quad x_i \leq z_{ix} k_{ix}^{\alpha} l_{ix}^{1-\alpha} \right\}$$

implying:

$$\begin{aligned} \frac{r_i}{p_{xi}} &= z_{ix} \alpha k_{ix}^{\alpha-1} l_{ix}^{1-\alpha} \\ \frac{w_i}{p_{xi}} &= z_{ix} (1-\alpha) k_{ix}^{\alpha} l_{ix}^{-\alpha} \end{aligned}$$

(b) Using the first order conditions, express the capital-labor ratio of each sector as a function of the ratio of the wage rate and rental rate of capital (w_i/r_i).

Using some algebra, from the foc:

$$\frac{r_i}{w_i} = \frac{\alpha}{1-\alpha} \frac{l_{ix}}{k_{ix}}$$

But then

$$\begin{aligned} \frac{k_{ix}}{l_{ix}} &= \frac{\alpha}{1-\alpha} \frac{w_i}{r_i} \\ \frac{k_{iy}}{l_{iy}} &= \frac{\beta}{1-\beta} \frac{w_i}{r_i} \end{aligned}$$

(c) Under the same assumptions what are the equilibrium price ratios p_{xi}/p_{yi}?

From foc:

$$\begin{aligned}\frac{p_{xi}}{p_{yi}} &= \frac{z_{iy}}{z_{ix}}\frac{\beta}{\alpha}\frac{\beta^{\beta-1}(1-\beta)^{1-\beta}}{\alpha^{\alpha-1}(1-\alpha)^{1-\alpha}}\left(\frac{w_i}{r_i}\right)^{\beta-\alpha} \\ &= \left(\frac{z_{iy}}{z_{ix}}\right)\cdot\left(\frac{w_i}{r_i}\right)^{\beta-\alpha}\cdot\frac{\beta^{\beta}(1-\beta)^{1-\beta}}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}}\end{aligned}$$

(d) Based on the expression in the previous part explain how this model has both, Ricardian and Hecksher-Ohlin, forces to explain the differences in comparative advantage?

Note how the first term on the RHS is related with Ricardo (higher productivity implies lower prices), while the second term is related with H-O (more labor, implies lower wages, which implies lower price of $x$if x is more labor intensive: $\beta > \alpha$).

(e) Under free trade, will factor price equalization hold (suppose $p_x$and p_y are the final price goods)? Why or why not?

Under international trade $p_{xf}/p_{yf} = p_{xh}/p_{yh} = p_x/p_y$ that is the LHS of the previous equation is constant. If $z_{yf}/z_{xf} = z_{yh}/z_{xh} = z_y/z_x$ that would imply that the ratio w/r had to be equal to both countries, that is, factor price equalization holds. However, when $z_{yf}/z_{xf} \neq z_{yh}/z_{xh}$ then factor price equalization doesn't hold and we no longer have $w_f/r_f = w_h/r_h$.

4. (15) Consider a 2 country version of the Armington model where consumption utility is given by:

$$\begin{aligned}C_h &= \left[\alpha^{1/\sigma}x_{hh}^{(\sigma-1)/\sigma} + (1-\alpha)^{1/\sigma}x_{fh}^{(\sigma-1)/\sigma}\right]^{\sigma/(\sigma-1)} \\ C_f &= \left[\beta^{1/\sigma}x_{ff}^{(\sigma-1)/\sigma} + (1-\beta)^{1/\sigma}x_{hf}^{(\sigma-1)/\sigma}\right]^{\sigma/(\sigma-1)}\end{aligned}$$

This world economy has transportation costs $\tau_{ij} \geq 1$ implying that prices are given by:

$$p_{ij} = w_i\tau_{ij}$$

for $i, j = h, f$, implying that p_{ij} is the destination price in country j of a good produced in country i. Moreover each country is endowed with a fixed ammount of labor that is used in production in the following sense:

$$\begin{aligned} L_h &= \tau_{hh} x_{hh} + \tau_{hf} x_{hf} \\ L_f &= \tau_{ff} x_{ff} + \tau_{fh} x_{fh} \end{aligned}$$

(a) Please derive a gravity equation for this economy.

Starting from the typical demand equation:

$$p_{hf} x_{hf} = (1-\beta) \left(\frac{p_{hf}}{P_f} \right)^{1-\sigma} \cdot X_f$$

Now, market clearing implies:

$$\begin{aligned} X_h &= w_h L_h \\ &= w_h \left(\tau_{hh} x_{hh} + \tau_{hf} x_{hf} \right) \\ &= p_{hh} x_{hh} + p_{hf} x_{hf} \\ &= \alpha \left(\frac{p_{hh}}{P_h} \right)^{1-\sigma} X_h + (1-\beta) \left(\frac{p_{hf}}{P_f} \right)^{1-\sigma} X_f \\ &= w_h^{1-\sigma} \left(\alpha \left(\frac{\tau_{hh}}{P_h} \right)^{1-\sigma} X_h + (1-\beta) \left(\frac{\tau_{hf}}{P_f} \right)^{1-\sigma} X_f \right) \end{aligned}$$

Finally, we can substitute this expression on the demand equation to get:

$$\begin{aligned} p_{hf} x_{hf} &= (1-\beta) \left(\frac{p_{hf}}{P_f} \right)^{1-\sigma} \cdot X_f \\ &= (1-\beta)\, w_h^{1-\sigma} \left(\frac{\tau_{hf}}{P_f} \right)^{1-\sigma} \cdot X_f \\ &= \left(\frac{\tau_{hf}}{P_f} \right)^{1-\sigma} \cdot X_f X_h \cdot \frac{(1-\beta)}{\alpha \left(\frac{\tau_{hh}}{P_h} \right)^{1-\sigma} X_h + (1-\beta) \left(\frac{\tau_{hf}}{P_f} \right)^{1-\sigma} X_f} \end{aligned}$$

(b) Why would a 1% increase in the income of the foreign economy (X_f) lead to a less than a 1% increase in the value of exports of the home economy ($p_{hf} x_{hf}$)? (recall that there may be some general equilibrium effects)

You can see that X_f shows up both in the numerator and denominator of the gravity equation so a 1% increase leads to a less than 1% increase in $p_{hf} x_{hf}$. The reason is related with the fact that higher demand for home's production of x_{hf}

leads to an increase in wages w_h that, in turn, reduces the ability of the home country to export.