# Final Exam - Solution Topics

International Trade I at ITAM

Professor: Tiago Tavares

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**Instructions:** Write your name and *clave* on the first page of your answer booklet/sheets and on the question booklet/sheets. Number all the sheets carefully, and staple them with the question sheets. You have 2 hours and 45 minutes to finish the exam. All questions have to be answered. Best of luck!

1. (30) Let the market for agricultural goods of a small economy be characterized by the following supply and demand functions:

$$\begin{aligned} Y^S &= 20 + 10P \\ Y^D &= 110 - 20P \end{aligned}$$

At the same time, the rest of the world which can be considered a much larger than the domestic one, can supply any amount of agricultural goods for a price of 2.

(a) Under free-trade how much will the domestic economy export or import? What are the equilibrium prices and quantities under autarky?
The import curve is given by:

$$IMP = Y^D - Y^S = 110 - 20P - 20 - 10P = 90 - 30P$$

Which, for $P = 2$, implies $IMP = 90 - 30 \times 2 = 30$.

(b) Now suppose that authorities in the domestic economy, in order to protect farmers from international competition decide to impose a price support to domestic agricultural goods of 4. That is, the government is now willing to buy all the required output to maintain prices at 4. Assume that the products purchased by

the government are destroyed. Under these conditions, how much will domestic consumers demand and how much will producers supply? What is the cost of this policy to the government?
With a price support of 4:

$$
\begin{aligned}
Y^S &= 20 + 10 \times 4 = 60 \\
Y^D &= 110 - 20 \times 4 = 30
\end{aligned}
$$

Because the government keeps the difference, the cost of this policy for the government becomes:

$$\Delta W^G = 30 \times 4 = 120$$

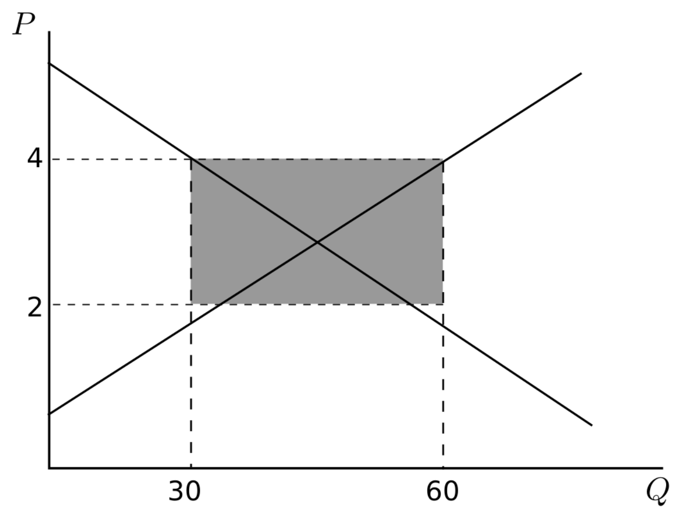
(c) Now, instead of destroying the goods that are purchased, the government resell those same goods to the world market at world prices. How much will the country export now? What is now the cost of the government policy?
Clearly that now:

$$\Delta W^G = 30 \times (4 - 2) = 60$$

(d) Comparing c) with a) [the free trade scenario], what is the net gain for society? Please also show your answer in a supply and demand diagram.
Note that:

$$
\begin{aligned}
\Delta W^C &= (2 - 4) \times \frac{((110.0 - 20.0 * 2.0) + (110.0 - 20.0 * 4.0))}{2} = -100 \\
\Delta W^P &= (4 - 2) \times \frac{((20.0 + 10.0 * 2.0) + (20.0 + 10.0 * 4.0))}{2} = 100 \\
\Delta W^G &= 30 \times (2 - 4) = -60 \\
\Delta W^S &= \Delta W^C + \Delta W^P + \Delta W^G = -60
\end{aligned}
$$

So, society is at a loss of -60 with this policy. That is the shaded region in the diagram:




(e) This type of policy is actually implemented in some developed economies. Would you support a similar policy for Mexico? (be brief in your answer)
I wouldn't support such a policy as it is highly distortionary to point of shifting an initial importing equilibrium to an exporting one. In other words, this policy is a disguised export subsidy where farmers benefit at the expense of domestic consumers and taxpayers (not to mention foreign countries' agents).

2. (45) Consider a Melitz model of trade with monopolistic competitive firms and two identical countries. Each firm produces a unique variety. Suppose a representative consumer values consumption accordingly to:

$$U = \left( \int_{\omega \in \Omega} q\left(\omega\right)^{\frac{\sigma-1}{\sigma}} d\omega \right)^{\frac{\sigma}{\sigma-1}}$$

where $q(\omega)$ in the consumption level of variety $\omega$ which has a price of $p\left(\omega\right)$. This consumer is endowed with $L$ units of labor with a respective wage rate of $w$.

(a) Set up the consumer utility maximization problem or, equivalently, the consumer expenditure minimization problem and derive the following demand function:

$$q(\omega) = \left( \frac{p\left(\omega\right)}{P} \right)^{-\sigma} Q$$

where

$$\begin{aligned} P &= \left( \int_{\omega \in \Omega} p(\omega)^{1-\sigma} \, d\omega \right)^{1/(1-\sigma)} \\ PQ &= wL \end{aligned}$$

The problem for the consumer is to minimize expenditure subject to some utility level:

$$\min_{q(\omega)} \left\{ \int_{\omega} p(\omega) q(\omega) \quad st \quad \left( \int_{\omega \in \Omega} q(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right)^{\frac{\sigma}{\sigma-1}} \geq Q \right\}$$

First order conditions gives:

$$p(\omega) = \lambda q(\omega)^{-1/\sigma} U^{1/\sigma}$$

Now, lets make use of the fact that $\lambda = P$ and $U = Q$ (and rearranging the above expression) to get:

$$q(\omega) = \left( \frac{p(\omega)}{P} \right)^{-\sigma} Q$$

(b) Suppose now that firms can produce to domestic or exporting markets where the unit labor requirements to produce one unit of output $q$ are, respectively, the following:

$$\begin{aligned} l_d(\varphi) &= f_d + q/\varphi \\ l_x(\varphi) &= f_x + q/\varphi \end{aligned}$$

this, for a firm with productivity $\varphi$. Additionally, if a firm wants to export, it needs to charge a $\tau > 1$ to cover for transportation costs. Please show that the optimal price charged by these monopolists is the following:

$$\begin{aligned} p_d(\varphi) &= \frac{\sigma}{\sigma - 1} \cdot \frac{1}{\varphi} \\ p_x(\varphi) &= \frac{\sigma}{\sigma - 1} \cdot \frac{\tau}{\varphi} \end{aligned}$$

where $p_d(\varphi)$ is the price for domestically produced goods while $p_x(\varphi)$ is the price for goods produced abroad. [normalize wages to $w = 1$]

Firms maximize their profits by choosing $p$ which equal ($i = x, d$):

$$\begin{aligned}\pi_i &= pq - l \\ &= pq - (f_i + q/\varphi) \\ &= p\left(\frac{p}{P}\right)^{-\sigma} Q - \left(f_i + \left(\frac{p}{P}\right)^{-\sigma} Q/\varphi\right)\end{aligned}$$

First order conditions and rearranging gives:

$$p_d(\varphi) = \frac{\sigma}{\sigma - 1} \cdot \frac{1}{\varphi}$$

And, from the definition of iceberg cost:

$$p_x(\varphi) = \frac{\sigma}{\sigma - 1} \cdot \frac{\tau}{\varphi}$$

(c) Derive the firms revenue functions, defined as $r(\varphi) = p(\varphi) \cdot q(\varphi)$, by arriving to the following result:

$$\begin{aligned}r_d(\varphi) &= \left(\frac{\sigma - 1}{\sigma}\varphi P\right)^{\sigma - 1} R \\ r_x(\varphi) &= \left(\frac{\sigma - 1}{\sigma}\varphi P\right)^{\sigma - 1} \tau^{1-\sigma} R\end{aligned}$$

where $R = PQ$, and $r_d(\varphi)$ is revenue that arrives from selling in the domestic market while $r_x(\varphi)$ is revenue that arrives from selling in the foreign market. Note that $r_i(\varphi) = q(\varphi)\, p(\varphi)$, then:

$$\begin{aligned}r_i(\varphi) &= q_i(\varphi)\, p_i(\varphi) \\ &= \left(\frac{p_i(\varphi)}{P}\right)^{-\sigma} Q p_i(\varphi) \cdot \frac{P}{P} \\ &= \left(\frac{p_i(\varphi)}{P}\right)^{1-\sigma} R \\ &= \left([p_i(\varphi)]^{-1} P\right)^{\sigma - 1} R\end{aligned}$$

Substituting with $p_i(\varphi)$:

$$\begin{aligned} r_d(\varphi) &= \left(\frac{\sigma-1}{\sigma}\varphi P\right)^{\sigma-1} R \\ r_x(\varphi) &= \left(\frac{\sigma-1}{\sigma}\varphi P\right)^{\sigma-1} \tau^{1-\sigma} R \end{aligned}$$

(d) Show that the profits functions can be written as:

$$\begin{aligned} \pi_d(\varphi) &= r_d(\varphi)/\sigma - f_d = & R\left(\frac{\sigma-1}{\sigma}\varphi P\right)^{\sigma-1} /\sigma - f_d \\ \pi_x(\varphi) &= r_x(\varphi)/\sigma - f_x = & R\left(\frac{\sigma-1}{\sigma}\varphi P\right)^{\sigma-1} \tau^{1-\sigma}/\sigma - f_x \end{aligned}$$

Please plot also these profit functions in a $(\pi, \varphi^{\sigma-1})$ diagram.
Note that:

$$\begin{aligned} \pi_i(\varphi) &= r_i(\varphi) - l_i(\varphi) \\ &= r_i(\varphi) - f_i - q_i(\varphi)/\varphi \\ &= r_i(\varphi) - f_i - r_i(\varphi)/(p(\varphi)\varphi) \\ &= r_i(\varphi) - f_i - r_i(\varphi)\frac{\sigma-1}{\sigma} \\ &= r_i(\varphi)\left(1 - \frac{\sigma-1}{\sigma}\right) - f_i \\ &= r_i(\varphi)/\sigma - f_i \end{aligned}$$

Therefore

$$\begin{aligned} \pi_d(\varphi) &= r_d(\varphi)/\sigma - f_d = & R\left(\frac{\sigma-1}{\sigma}\varphi P\right)^{\sigma-1} /\sigma - f_d \\ \pi_x(\varphi) &= r_x(\varphi)/\sigma - f_x = & R\left(\frac{\sigma-1}{\sigma}\varphi P\right)^{\sigma-1} \tau^{1-\sigma}/\sigma - f_x \end{aligned}$$

(e) Using the above profit functions we can show that, under some general conditions, there will be 2 cut-off productivities ($\varphi_d^*$ and $\varphi_x^*$) characterized by $\pi_d(\varphi_d^*) = 0$ and $\pi_x(\varphi_d^*) = 0$ and that determine whether a firm produces only for the domestic market or whether is also produces to the foreign market. Using this fact, show that both cut-offs are characterized by:

$$\varphi_x^* = \tau \left(\frac{f_x}{f_d}\right)^{\frac{1}{\sigma-1}} \varphi_d^*$$

Note that $\pi_d(\varphi_d^*) = 0$ and $\pi_x(\varphi_d^*) = 0$ imply:

$$\begin{aligned} R\left(\frac{\sigma-1}{\sigma}\varphi P\right)^{\sigma-1} &= f_d\sigma \\ R\left(\frac{\sigma-1}{\sigma}\varphi P\right)^{\sigma-1}\tau^{1-\sigma} &= f_x\sigma \end{aligned}$$

Substituting the first equation into the second one and rearranging:

$$\varphi_x^* = \tau \left(\frac{f_x}{f_d}\right)^{\frac{1}{\sigma-1}} \varphi_d^*$$

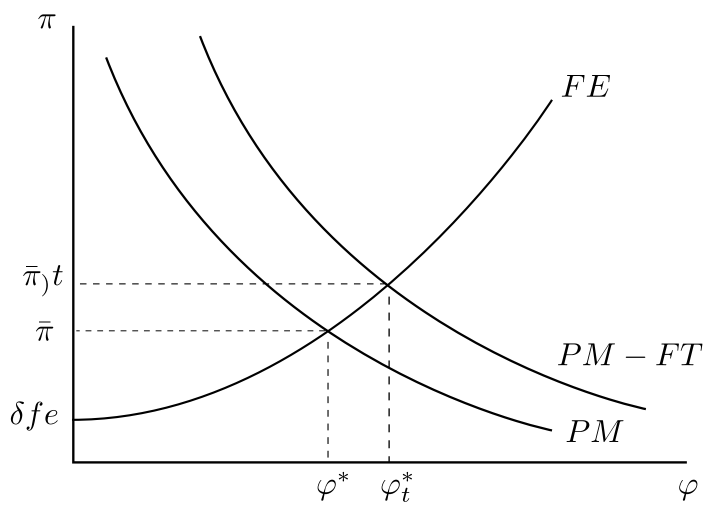
(f) Once we add a standard entry mechanism to close the model, we can show that the equilibrium can be characterized by a free-entry condition and a zero-profit condition, namely:

$$\bar{\pi} = \frac{\delta f_e}{1 - G\left(\varphi_d^*\right)} \tag{FE}$$

$$\bar{\pi} = f_d \left[\left(\frac{\tilde{\varphi}_d\left(\varphi_d^*\right)}{\varphi_d^*}\right)^{\sigma-1} - 1\right] + \frac{1 - G\left(\varphi_x^*\left(\varphi_d^*\right)\right)}{1 - G\left(\varphi_d^*\right)} \cdot f_x \cdot \left[\left(\frac{\tilde{\varphi}_x\left(\varphi_d^*\right)}{\varphi_x^*\left(\varphi_d^*\right)}\right)^{\sigma-1} - 1\right] \tag{ZP}$$

where $\delta$ is a period-to-period dying probability, $G(\varphi)$ is the exogenous distribution function of firms productivities, $\tilde{\varphi}_d$ is the average productivity of surviving firms that produce in domestic markets, and $\tilde{\varphi}_x$ is the average productivity of surviving firms that produce in foreign markets. Use the diagrams that you've learn in classes to analyze how the equilibrium changes when $f_d$ - the fixed cost of producing to domestic markets - increase. In your answer be specific to mention what happens in terms of firm selecting into domestic and export markets, in particular, about how the distance $\varphi_d^*$ and $\varphi_x^*$ changes.

It is clear that the first equation (FE) doesn't change with $f_d$. As for the second equation (ZP), the first term increases with $f_d$, while the second term is not so clear. Let's assume that it also increases with $f_d$. Then a new equilibrium follows with higher $\varphi_d^*$ and $\bar{\pi}$:



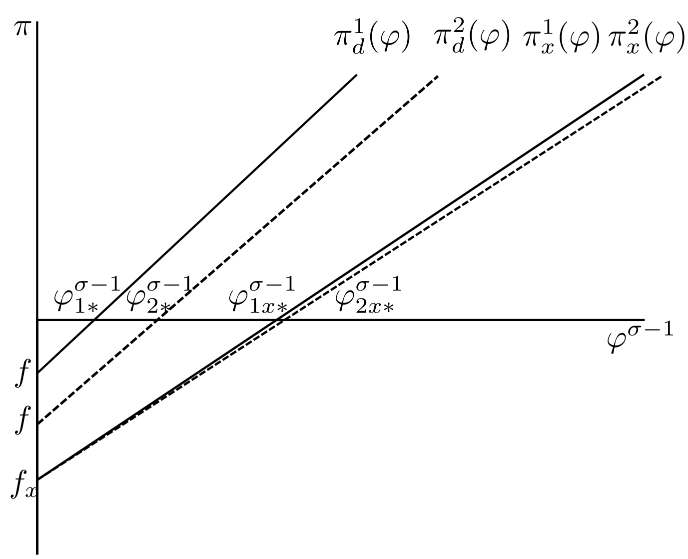

Regarding selection, note that

$$\frac{\varphi_x^*}{\varphi_d^*} = \tau \left(\frac{f_x}{f_d}\right)^{\frac{1}{\sigma-1}}$$

or

$$\left(\frac{\varphi_x^*}{\varphi_d^*}\right)^{\sigma-1} = \tau^{\sigma-1}\frac{f_x}{f_d}$$

It follows that when $f_d$ increases, the ratio $\varphi_x^*/\varphi_d^*$ decreases. It follows that a possible change in the profit functions is the following:



That is: now, only more productive firms can operate in the domestic markets, this can increase the productivity of firms that operate in the foreign markets, at the same time the productivity range of firms operating exclusively in the domestic market becomes more narrow.

(g) Vary briefly, list and explain 2 empirical features of the data that can be explained by the Melitz model but not by the previous models of trade that we've studied in this class.

Note that in the Melitz model only a small set of firms operating in country engage in international trade, at the same time, those firms are the most productive ones. This actually fit the empirical observations that exporting is a rare activity and that exporting firms are the most productive ones (larger size in terms of employment and higher revenues per worker).

3. (25) Imagine a small open economy that produces two main goods: agricultural and manufacturing goods. We assume that the agricultural sector in the economy uses labor intensively in comparison with the manufacturing goods that uses more capital. Firms in both sectors operate in a perfect competition environment and the country is endowed with a particular level of capital and labor.

(a) Can you predict what will happen to the real returns on capital and labor if the economy experiences a terms of trade shock that increases the relative price manufacturing goods? Who will be the winners and who will be the losers of shift in terms of trade?

Note that this is an application of the Stolper-Samuelson theorem. The price of manufacturing rises relatively to the price of agriculture. Because the manufactory sector uses capital intensively, the real return on capital will increase relatively to the real return on labor. This implies that works will loose while capitalists will win from this terms-of-trade shock.

(b) What would be your forecast of the economic impacts of a new immigration policy that has as a goal the increase the total labor supply of the economy?

Now we have an application of the Rybczynsky theorem. Because the labor endowment increases in the country, the agricultural sector that uses labor intensively will increase its output, while the manufacturing sector will decrease it.

(c) Please state in words the Rybczynsky theorem (you don't need to be formal, but just to express correctly the main idea of the theorem).

An increase in factor endowment will increase the output of the industry using it intensively, and decrease the output of the other industry